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Source: *The RAND Journal of Economics*, Vol. 22, No. 1 (Spring, 1991), pp. 89-106
Published by: Wiley on behalf of RAND Corporation
Stable URL: https://www.jstor.org/stable/2601009
Accessed: 03-01-2020 13:12 UTC

RAND Journal of Economics
Vol. 22, No. 1, Spring 1991

The impact of cyclical demand movements on collusive behavior

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Recent work by Rotemberg and Saloner (1986) investigates the effect of the business cycle on optimal collusive pricing by specifying that demand is subject to i.i.d. shocks. An implication of the i.i.d. assumption is that firms' expectations on future demand are unrelated to the current level of demand. We put forth a model that allows for both the level of current demand and firms' expectations on future demand to change over time; thus it captures two important properties of the business cycle. Our analysis reveals that while the gain to deviating from a collusive agreement is greatest during booms, firms find it most difficult to collude during recessions, as the forgone profits from inducing a price war are relatively low. An implication of this effect for pricing behavior is that at the same level of demand, price is lower when demand is declining than when demand is rising. Consistent with previous theoretical work, we find that firms price countercyclically for a range of values for the discount factor. However, numerical simulations reveal a greater tendency for firms to price countercyclically during recessions than during booms.

1. Introduction

■ How the business cycle affects collusive pricing has long been an issue of interest in industrial organization and macroeconomics. Related to this issue are at least two fundamental questions. First, is it more difficult for firms to collude during recessions or during booms? Based largely upon pre–World War II case studies, it has long been believed that collusion is more difficult during economic downturns. This position is weakly supported by Suslow (1988); constructing a data set from many of these case studies, she found the probability of breakdown of a cartel to be indeed higher during recessions.[1] A second issue

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The authors are grateful for the helpful comments and suggestions of Tim Bresnahan, Myong Chang, Luke Froeb, Gregg Hess, Jim Hess, Michi Kandori, Jim Leitzel, two anonymous referees, seminar participants at Johns Hopkins, the UNC-Duke Theory Workshop, the Department of Justice, North Carolina State, and the University of Southern California, and conference participants at the 1988 Carleton Industrial Organization Conference, the 1988 Southern Economic Association Meetings, and the 1988 NBER Summer Institute.

[1] Additional evidence in support of the hypothesis that it is tougher to collude during recessions is provided by Montgomery (1987). He finds that cartel agreements in the transatlantic steerage trade were most often breaking down after the peak of a boom and prior to the trough of a recession.

is: What is the relationship between price movements and fluctuations in market demand? There have been several empirical studies along this line. In reviewing the empirical literature, one finds that the evidence is mixed in that there is empirical verification of both procyclical and countercyclical pricing.[2]

In light of the either scant or mixed empirical evidence on oligopolistic pricing over the business cycle, it is essential to improve our understanding of the incentives faced by firms in setting price when demand fluctuates over time. The development of formal theories should lead to improved empirical testing. Toward this end, recent work by Rotemberg and Saloner (1986) explores optimal collusive pricing when demand is subject to (observable) i.i.d. shocks. Their analysis revealed, in contrast to the traditional view, that it is generally more difficult for firms to collude during booms where a boom is defined as when the level of market demand is high. The rationale is that the gain from cheating on a collusive agreement is highest when current demand is strongest. Even more striking, they found it may be optimal for firms to price countercylically in order to maintain the stability of the collusive arrangement.

The analysis of Rotemberg and Saloner (1986) is an insightful beginning toward understanding the relationship between collusive pricing and the business cycle. The next step in this line of research is to enrich the environment to allow for further properties of the business cycle. In particular, by specifying that demand shocks are i.i.d., the Rotemberg and Saloner model leaves out an important element of the business cycle that may have significant implications for collusive pricing. A result of the i.i.d. assumption is that firms' expectations on future demand are independent of current demand, and one implication of this is that firms never expect demand to be stronger tomorrow if it is relatively strong today. This is unfortunate if one is concerned with understanding the influence of the business cycle on pricing behavior, since "stronger demand tomorrow" is exactly what firms' expectations will be if they believe the economy is in an upturn. To see the potential sensitivity of Rotemberg and Saloner's results to the i.i.d. assumption, note that anticipations of future demand alter the incentives to cheat on a collusive agreement. If strong demand today signals strong demand tomorrow, firms would be forgoing high collusive profits in the immediate future (when demand is expected to be strong) by cheating today on a collusive agreement. Thus, it is no longer clear that higher current demand makes collusion more difficult when firms' expectations on future demand are related to the state of current demand. While the analysis of Rotemberg and Saloner has allowed for one effect of the business cycle—that demand changes over time—it has not taken account of an equally important effect: that firms' expectations on future demand also change over time.[3]

This article develops a model that allows for both features of the business cycle by specifying that the market demand function moves cyclically over time. In this manner, both current demand and firms' expectations on future demand are allowed to change over time. Although demand movements are specified to be deterministic, the model is such that results are robust to allowing for (nonobservable) i.i.d. demand shocks. In that case it is firms' expectations on future demand that move cyclically.

Our objective for investigating this model is twofold. First, we wish to assess the robustness of Rotemberg and Saloner's findings when firms' expectations on future demand move over time. Concerning their result that it is toughest to collude during booms, it is important to identify the difference between a boom (or a recession) in Rotemberg and Saloner's framework and in our framework. Since Rotemberg and Saloner only allow for the level of demand to change over time, a boom (recession) is when the level of demand

[2] See, for example, Bils (1987a), Domowitz, Hubbard, and Petersen (1986a, 1986b, 1987), and Rotemberg and Saloner (1986).

[3] See Zarnowitz (1985) for evidence of the serial correlation exhibited in business cycles that motivates this discussion.

is abnormally high (low). In contrast, since our framework also allows the expected direction of demand to change over time, a boom (recession) is when demand is increasing (decreasing). Our formulation is consistent with the standard (e.g., NBER reference cycle) chronology of booms and recessions. Our analysis shows that the most difficult point of the cycle for firms to collude is not necessarily when demand is high but rather when demand is falling. Hence, in support of the traditional viewpoint that emerged from the industry case studies, firms find it toughest to collude during recessions. While this finding of Rotemberg and Saloner is found to be sensitive to the i.i.d. assumption, their result that countercyclical pricing is optimal for some values of the discount factor is strongly supported. Independent work by Kandori (1988) also supports the robustness of countercyclical pricing and will be discussed later in the article.

A second reason for exploring this model is that the preceding theoretical and empirical literature on pricing over the business cycle has focused solely upon very general properties: specifically, whether pricing is pro- or countercyclical. Our study expands the range of the analysis to consider asymmetries in collusive pricing: specifically, properties that depend on whether the market is in an upturn or a downturn. Analytically we find that prices are lower when demand is declining than when demand is rising, *ceteris paribus,* while numerical simulations suggest that firms are more likely to price countercyclically during downturns than during upturns. These results are consistent with it being more difficult for firms to collude during recessions.

The plan of the article is as follows. After we present the model in Section 2, in Section 3 we set out a predictive theory of collusive pricing and derive properties of the collusive price and profit path when demand is subject to cyclical movements. This analysis includes a characterization of when it is most difficult for firms to collude and the resulting implications for price and profit behavior over the cycle. Numerical simulations of the optimal collusive pricing path are provided in Section 4 to bring additional insight to the analysis. We offer concluding remarks in Section 5. All proofs are relegated to the Appendix.

2. The model

■ Consider an industry with n firms where $n \geq 2$ and finite. Firms are infinitely lived and are modelled as interacting in a Bertrand price game in each period. Hence, in each period firms will simultaneously choose price. Symmetry is assumed as firms offer homogeneous products and have identical cost functions of the form $C(q) = cq$, where $c \geq 0$. Market demand in period t is represented by $D(P; t)$ and is specified to have the following properties.

Assumption 1. $D(\cdot\,; t)$: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a continuous, bounded function, $\forall t$.

Assumption 2. There exists $\bar{P}(t) > c$ such that $D(P) = 0$ if and only if $P \geq \bar{P}(t)$, $\forall t$.

Assumption 3. $D(\cdot\,; t)$ is decreasing in $P\ \forall P \in [0, \bar{P}(t)]$, $\forall t$.

As is standard, we assume market demand is equally divided among the lowest-priced firms. Finally, we assume that the joint profit function is strictly quasi-concave in price.

Assumption 4. $(P - c)D(P; t)$ is strictly quasi-concave in $P\ \forall P \in [0, \bar{P}(t)]$, $\forall t$.

An implication of Assumption 4 is that there exists a unique monopoly price $P^m(t)$.

To investigate the impact of cyclical fluctuations on collusive pricing, a particular structure must be placed upon the intertemporal movement of market demand. For this purpose, define $\bar{t}$ as the number of periods in one cycle, that is, the minimum number of periods such that the time path of demand begins to repeat itself. We assume $\bar{t}$ to be finite. The market demand function then follows the time path in (1).

$$D(P;t) = \begin{cases} D(P;1) & \text{if} \quad t \in \{1, \bar{t}+1, 2\bar{t}+1, \ldots\}, \\ D(P;2) & \text{if} \quad t \in \{2, \bar{t}+2, 2\bar{t}+2, \ldots\}, \\ \vdots & \qquad \vdots \\ D(P;\bar{t}) & \text{if} \quad t \in \{\bar{t}, 2\bar{t}, 3\bar{t}, \ldots\}. \end{cases} \tag{1}$$

The only restriction we will place on this cycle is that it be single-peaked. That is, starting at period 1 of the cycle, demand is assumed to shift out over time up to some period $\hat{t}$, where $\hat{t} \in \{2, \ldots, \bar{t}\}$, at which point demand will shift back in until it reaches its minimum level of $D(P; 1)$ at $\bar{t} + 1$. Letting $D(P; t') \gg D(P; t'')$ denote $\bar{P}(t') \geq \bar{P}(t'')$ and $D(P; t') > D(P; t'')\ \forall P \in (0, \bar{P}(t'))$, this single-peaked condition is embodied in Assumption 5.[4]

Assumption 5. $D(P; 1) \ll D(P; 2) \ll \ldots \ll D(P; \hat{t}) \gg D(P; \hat{t} + 1) \gg \ldots D(P; \bar{t}) \gg D(P; 1)$.

A direct implication of Assumption 5 is that the industry profit function moves in the same direction as market demand over time. The industry profit function shifts up from period 1 to period $\hat{t}$, reaches its peak at $\hat{t}$, and then shifts down from period $t + 1$ to $\bar{t} + 1$.[5]

Other than the restriction that the cycle be single-peaked, no conditions are placed upon the demand cycle. The length of a boom can be greater, the same, or smaller than the length of a recession. Similarly, the speed at which the market recovers from a recession can be greater, the same, or smaller than the speed at which it moves from a boom into a recession. It is important to note that while the motivation for our assumptions on demand is the business cycle, our model is also applicable to other sources of cyclical fluctuations, including seasonal cycles and fads.

Given this demand and cost structure, in each period firms make simultaneous price decisions and supply so as to meet demand. (As long as $P_i^t \geq c$, which will indeed be true in equilibrium, this is an optimal supply response for firm i.) With an infinite horizon, a strategy for firm i is an infinite sequence of action functions, $\{S_i^t\}_{t=1}^{\infty}$, where the period t action function maps from the set of possible histories to the game (a history being the past prices of the n firms) into the set of possible prices that firm i can choose. The payoff function for firm i is the sum of discounted profits where the common discount factor is $\delta \in (0, 1)$.

In concluding the description of our model, let us briefly discuss two important assumptions. These are that market demand moves deterministically and that market structure is exogenously fixed. With respect to the latter assumption, we are implicitly assuming that cyclical fluctuations are not so severe as to induce entry or exit. As noted by Scherer (1980), this assumption is reasonable for many manufacturing industries.[6] Those industries for

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[4] This assumed pattern of demand can be motivated as being consistent with either endogenous business cycle models or seasonal cycle models. It also provides insight into effects that would be present if demand is stochastic but serially correlated. In this regard, the particular patterns of demand that we consider are, in general, not consistent with linear, first-order serial correlation models but rather with higher-order and potentially nonlinear serial correlation processes.

[5] Actually, the crucial assumption for our ensuing results is that the industry profit function, rather than the market demand function, moves in a cyclical manner. For expositional purposes, we found it easier to assume cyclical movements in market demand. An alternative way to generate such a pattern in the profit function is to assume that demand is fixed and marginal cost is countercyclical—though such an assumption appears contrary to the empirical findings of Bils (1987a). On the other hand, one could assume that marginal cost is procyclical but also that market demand is sufficiently procyclical so that the profit function is procyclical.

[6] Concerning the assumption that demand fluctuations are not so severe as to cause changes in industry structure, let us quote Scherer (1980):

which entry and exit is an important cyclical phenomenon, one will have to adapt this model to allow market structure to be endogenous.[7] With regard to the assumption that movements in market demand are fully anticipated, it is significant to note that our model is equivalent to one in which demand follows a cyclical pattern but is also subject to non-observable i.i.d. shocks. In that case, firms are uncertain of future demand but anticipate the cyclical movement in the probability distribution over demand. When nonobservable i.i.d. shocks are allowed, $D(P; t)$ is then interpreted as firms' expectation of market demand in period t, given a price P. With this interpretation and risk-neutral firms, the ensuing analysis applies.

3. Prices and profits over the business cycle

■ This section puts forth a predictive theory of collusive pricing and derives basic properties of the collusive price and profit path when the market demand function is subject to cyclic fluctuations. We then consider the issue of when during the business cycle it is most difficult for firms to collude, and we derive implications for the behavior of prices over the cycle.

□ **Theory of collusive pricing.** In order to have a predictive theory of collusive pricing, we use the commonly employed specification that the n (identical) firms select a symmetric price path so as to maximize their joint payoffs, subject to the constraint that the price path be supportable by a subgame perfect equilibrium.

The first step is to characterize the set of price paths that are supportable by subgame perfect equilibria. Since a grim trigger strategy profile results in a deviator receiving its minimax payoff of zero, it represents a most severe punishment strategy equilibrium. Therefore, a price path is supportable by subgame perfect equilibria if and only if it is supportable by grim trigger strategies. Given a collusive price path $\{P(t)\}_{t=1}^{\infty} \in \prod_{t=1}^{\infty} (c, P^m(t)]$, the associated profile of grim trigger strategies is of the form

$$S_i^1 = P(1),$$

$$S_i^t = \begin{cases} P(t) & \text{if} \quad P_j^\tau = P(\tau)\ \forall \tau \in \{1, \ldots, t-1\}, \forall j \in \{1, \ldots, n\}, \\ c & \text{otherwise}; \end{cases} \tag{2}$$

$$t \in \{2, 3, \ldots\}, i \in \{1, \ldots, n\}.$$

This strategy profile calls for each firm to initially price at the collusive level of $P(1)$ in period 1 and to continue to price according to $\{P(t)\}_{t=1}^{\infty}$ as long as no firm has deviated

It must be noted too that major bankruptcies are relatively rare. In 1975, at the trough of an unusually severe general business recession, there were 1,645 recorded manufacturing business failures, out of a total population of 450,000 incorporated and unincorporated manufacturing business enterprises. Nearly half of the failures had liabilities of less than \$100,000, and the total liabilities of failing manufacturers approximated \$1 billion.

Note, however, that more recent evidence (see, for example, Dunne, Roberts, and Samuelson (1989) and Davis and Haltiwanger (forthcoming)) reveals tremendous gross turnover of jobs, of which a substantial fraction is accounted for by plant entry and exit. Further, Davis and Haltiwanger (forthcoming) present evidence that gross job destruction due to plant exit rises substantially during business cycle slumps.

[7] For the case when firms do not collude, Chatterjee and Cooper (1988) examine industry behavior over a business cycle when market structure is endogenous. For the case when firms collude but demand is fixed over time, market structure is endogenously derived in MacLeod (1987), Mookherjee and Ray (1987), Paul (1988), and Harrington (1989).

from this path. If a firm does deviate, all firms revert to their single-period Nash equilibrium strategy of pricing at unit cost.[8]

It is straightforward to show that necessary and sufficient conditions for the strategy profile in (2) to form a subgame perfect equilibrium are

$$L(t; \{P(\tau)\}_{\tau=1}^{\infty}, \delta) \equiv \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}[(P(\tau) - c)(D(P(\tau); \tau)/n)]$$
$$\geq (n-1)(P(t) - c)(D(P(t); t)/n) \equiv G(t; \{P(\tau)\}_{\tau=1}^{\infty}), \forall t \geq 1. \quad (3)$$

$L(t; \{P(\tau)\}_{\tau=1}^{\infty}, \delta)$ is the discounted future loss from deviating in period t from the collusive price path, and it represents forgone future collusive profits. Since the optimal deviation policy in period t is to slightly undercut the collusive price $P(t)$ (and this results in an increase of $(n-1)(D(P(t); t)/n)$ in the deviator's demand), $G(t; \{P(\tau)\}_{\tau=1}^{\infty})$ is the one-time gain from cheating. Defining $\Delta(\delta)$ to be the set of price paths that are supportable by subgame perfect equilibria, it follows from (3) that

$$\Delta(\delta) \equiv \{\{P(\tau)\}_{\tau=1}^{\infty} \in \prod_{\tau=1}^{\infty} [c, P^m(\tau)] \mid L(t; \{P(\tau)\}_{\tau=1}^{\infty}, \delta)$$
$$\geq G(t; \{P(\tau)\}_{\tau=1}^{\infty}), \forall t \geq 1\}. \quad (4)$$

The optimization problem faced by n colluding firms is stated in Theorem 1. Theorem 1 shows that there always exists a solution to it.

Theorem 1. $\forall \delta \in (0, 1)$, there exists $\{P^*(t; \delta)\}_{t=1}^{\infty}$ such that

$$\sum_{t=1}^{\infty} \delta^t[(P^*(t; \delta) - c)(D(P^*(t; \delta); t)/n)]$$
$$= \max_{\{P(\tau)\}_{\tau=1}^{\infty} \in \Delta(\delta)} \sum_{t=1}^{\infty} \delta^t[(P(t) - c)(D(P(t); t)/n)]. \quad (5)$$

Proof. See the Appendix.

□ **The cyclic properties of prices.** Our first two results are standard. Theorem 2 shows that the joint profit-maximizing price path is sustainable if and only if the discount factor is sufficiently high. Theorem 3 shows that firms must price at the competitive level when the discount factor is sufficiently low.

Theorem 2. There exists $\hat{\delta} \in ((n-1)/n, 1)$ such that $\{P^*(t; \delta)\}_{t=1}^{\infty} = \{P^m(t)\}_{t=1}^{\infty}$ if and only if $\delta \in [\hat{\delta}, 1)$.

Proof. See the Appendix.

By Theorem 2, we know that when the discount factor is sufficiently high, the collusive price path will be procyclical if the intertemporal movement in market demand is such that the unconstrained joint profit-maximizing price path is procyclical. If instead $P^m(t)$ is countercyclical, then, when the discount factor is sufficiently high, firms are observed to price countercyclically. This latter possibility has been pointed out by Bils (1987b), who shows

[8] For ease of analysis, we focus upon infinite punishments. All results go through if instead punishment entails reversion to marginal cost pricing for T cycles, where T is finite.

that $P^m(t)$ is countercyclical if demand during boom periods is sufficiently elastic so that firms optimally reduce price in response to an upward shift in demand.[9]

Theorem 3. If $\delta \in (0, ((n-1)/n))$, then $P^*(t) = c \; \forall t$.

Proof. See the Appendix.

We have a more interesting analysis when the discount factor falls in an intermediate range: $\delta \in [((n-1)/n), \hat{\delta})$. To understand how the optimal collusive price path is determined in that case, let us pursue the thought experiment of lowering δ from $\hat{\delta}$. When δ is reduced slightly below $\hat{\delta}$, there will be some point of the cycle, which we will denote t^*, such that $P^m(t)$ is sustainable at all points of the cycle except t^*. Since there is an incentive to deviate at t^* if $P^m(t^*)$ is the collusive price, $P^*(t^*)$ must be less than $P^m(t^*)$ if cheating is to be made unprofitable. If at this lower price $P^m(t)$ is still sustainable at the other $\bar{t}-1$ points, then $P^*(t)$ equals $P^m(t)$ for all these points. The reason is as follows. Suppose firms set price below $P^m(t^0)$ for some $t^0 \in \{\{1, \ldots, \bar{t}\} - \{t^*\}\}$. Since profits at t^0 would be lower, this could only be optimal if it reduced the incentive to deviate at t^* so that price (and profits) at t^* could be raised. However, just the contrary occurs. Since profits are lower at t^0, the loss to cheating at other points of the cycle is reduced so that there is a greater incentive to deviate at t^*. It follows that it is optimal to set price at $P^m(t)$ whenever possible. As δ is reduced further, price will eventually have to be set below the joint profit-maximizing level, not only at t^* but also at additional points. However, at those points for which the joint profit-maximizing level is still sustainable, firms will optimally set price at that level according to the argument given above. Eventually, when δ is low enough, $P^*(t) < P^m(t)$ for all $t \in \{1, \ldots, \bar{t}\}$.

As described above, when δ is reduced below $\hat{\delta}$, the collusive price must be set below the joint profit-maximizing price at some point of the cycle. Theorem 4 shows that this point must be when demand is expected to fall.

Theorem 4. There exists $\tilde{\delta} \in (((n-1)/n), \hat{\delta})$ such that there exists $t^* \in \{\hat{t}, \ldots, \bar{t}\}$ such that $P^*(t^*) < P^m(t^*)$ and $P^*(t) = P^m(t) \; \forall t \in \{\{1, \ldots, \bar{t}\} - \{t^*\}\} \; \forall \delta \in [\tilde{\delta}, \hat{\delta})$. Furthermore, $P^*(t^*) < P^m(t^*) \; \forall \delta \in (0, \hat{\delta})$.

Proof. See the Appendix.

The method of proof of Theorem 4 is to show that for any point at which demand is rising (that is, $t \in \{1, \ldots, \hat{t}-1\}$), one can find a point at which demand is falling and that yields at least as high a one-time gain from cheating. The loss from cheating is higher at the point for which demand is rising, since, *ceteris paribus,* collusive profits over the immediate future are greater compared to when demand is falling. It follows that the incentive compatibility constraint is more likely to be binding at the point at which demand is falling than at the point at which demand is rising. Therefore, as we reduce δ below $\hat{\delta}$, the optimal collusive price will have to be set below the joint profit-maximizing price during a recessionary period rather than a boom period.[10]

If we define the toughest point of the cycle for firms to collude to be that point at which it is most difficult to sustain the joint profit-maximizing price, then Theorem 4 shows that the toughest point is during a recession. Intuitively, when demand is rising during a boom,

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[9] To provide an example in which $P^m(t)$ is countercyclical, consider the following two-point cycle. Let the market demand function during boom periods be $D(P) = P^{-\alpha}$, where $\alpha \in (1, 2)$, and during bust periods let it be $D(P) = 1 - P$. Note that $P^{-\alpha} > 1 - P \; \forall P > 0$. Given that the monopoly price during boom periods is $[(\alpha/(\alpha-1))c]$ and during bust periods is $[(1+c)/2]$, it is straightforward to show that $P^m(t)$ is countercyclical if and only if $c < [(\alpha-1)/2(2-\alpha)]$.

[10] Using a specific functional form for the deterministic business cycle, this result was derived independently by Montgomery (1988).

firms would expect to lose large profits in the immediate future if they were to induce a price war. Thus, firms have less to lose from undercutting the collusive price during a recession, so there is a greater incentive to deviate. To offset this incentive, price must be lowered. This result is to be contrasted with the finding of Rotemberg and Saloner (1986) that it is tougher to collude during booms. We shall discuss these two results later in this section.

While one might hope to be able to say more about the toughest point of the cycle for collusion, it is not possible given the general class of cycles being considered. For any $t \in \{\hat{t}, \ldots, \bar{t}\}$, one can always find a time path of demand consistent with our assumptions such that $t^* = t$ (see Haltiwanger and Harrington (1987)). It is then possible for the toughest point to be the peak of a boom or the last point before the trough of a recession or any point between the two.

Depending on where the discount factor lies in the interval $[(n-1)/n, \hat{\delta})$, the optimal collusive price path can have quite distinct properties. Since $P^*(t) = P^m(t)$ for all t when $\delta \geq \hat{\delta}$, it follows by continuity that if δ is less than but close to $\hat{\delta}$, then the price path is procyclical (when $P^m(t)$ is procyclical). However, when δ is close to $((n-1)/n)$, firms optimally price countercyclically. This is shown in Theorem 5.

Theorem 5. There exists $\bar{\delta} \in ((n-1)/n, \tilde{\delta})$ such that

$$P^*(1;\delta) > P^*(2;\delta) > \ldots > P^*(\hat{t};\delta) < P^*(\hat{t}+1;\delta) < \ldots < P^*(\bar{t};\delta) < P^*(1;\delta) \ \forall \delta \in [(n-1)/n, \bar{\delta}).$$

Proof. See the Appendix.

In proving Theorem 5, we first show that when $\delta = ((n-1)/n)$, the only sustainable collusive price paths are those that result in firm profits being flat over the cycle. In order to achieve this profit pattern, the stronger the demand, the lower the price that firms must set. Hence, we observe countercyclical pricing. A continuity argument implies that firms price countercyclically for all discount factors in the interval $[(n-1)/n, \bar{\delta})$.

That colluding firms optimally price countercyclically for some discount factors was originally derived by Rotemberg and Saloner (1986). While their model is quite distinct from ours, the forces responsible for this result are very much the same, in that when the discount factor is lowered, the price path must be set so as to reduce the fluctuations in industry profits. When industry profits fluctuate greatly, there is a relatively strong incentive to cheat in those periods for which profits are high, as the gain to cheating is high. As a result, when the discount factor is relatively low, the only sustainable price paths are those in which the fluctuations in profits are kept to a minimum. For this to be achieved, price must be set relatively high when demand is weak and relatively low when demand is strong so that firms price countercyclically. More evidence of the robustness of countercyclical pricing is found in the work of Kandori (1988). He extends the analysis of Rotemberg and Saloner (1986) to the case of serially correlated shocks and shows that when δ is near $((n-1)/n)$, firms price countercyclically. One is then led to conclude that Rotemberg and Saloner's finding—over a range of values for the discount factor, colluding firms price countercyclically—is quite robust to the way in which demand fluctuations are modelled.

□ **The cyclic properties of profits.** From the preceding analysis, one concludes that the collusive price path can take many different forms. If the unconstrained joint profit-maximizing price path is procyclical, the collusive price path can be procyclical (if δ is sufficiently high) or countercyclical (if δ is in the intermediate range $[(n-1)/n, \tilde{\delta})$). In contrast, general properties can be derived for the time path of profits. Theorem 6 shows that the collusive profit path is always single-peaked, with the peak occurring during the boom. $\pi^*(t)$ is defined to be the equilibrium level of industry profits:

$$\pi^*(t) \equiv (P^*(t; \delta) - c)D(P^*(t; \delta); t).$$

Theorem 6. $\forall \delta \in ((n-1)/n, 1), \exists \tilde{t} \in \{2, \ldots, \hat{t}\}$ such that

$$\pi^*(1) < \ldots < \pi^*(\tilde{t}) > \ldots > \pi^*(\bar{t}) > \pi^*(1) \quad \text{and}$$
$$\pi^*(t) = (P^m(t) - c)D(P^m(t); t)\ \forall t \in \{1, \ldots, \tilde{t} - 1\}.$$

Proof. See the Appendix.

According to Theorem 6, the industry earns monopoly profits during the early part of a boom, so that profits are rising. Eventually profits must decline, and once they do they will continue to decline over the remainder of the cycle. Industry profits weakly lead the cycle and thus decline throughout a recession.

Theorem 6 is a powerful result in that it provides a general property of the time path of profits in collusive industries. Intuitively, as the recession approaches, firms find it increasingly difficult to collude because the forgone profits from starting a price war diminish as the time at which demand is expected to fall grows nearer. To offset this effect, the collusive price must be lowered. Furthermore, it is lowered to such a degree that profits can begin to decline before the peak of the boom even though the market demand function is still shifting out. When the recession finally hits, profits continue to fall as the market demand function is shifting in, and price must still be set at a relatively low level because the incentive to cheat is still relatively great. As a result, we find that industry profits weakly lead the cycle in collusive industries. This result provides a testable implication of collusion, as industry profits always move with the cycle in noncollusive industries.

□ **Prices in booms versus recessions.** The preceding literature has focused upon properties of collusive pricing that are stable over the business cycle. In this subsection, we shall consider systematic effects of cyclic fluctuations on collusive pricing that change over the cycle. In light of our finding that firms find it most difficult to collude when demand is falling, we are particularly interested in determining whether the collusive price path differs systematically between recessions and booms.

Theorem 7 shows, controlling for the market demand function, that the collusive price during a boom is always at least as great as the collusive price during a recession.

Theorem 7. If $D(P; t') = D(P; t'')\ \forall P \geq 0$, where $t' < \hat{t} < t''$, then: (i) $P^*(t'; \delta) \geq P^*(t''; \delta)\ \forall \delta \in (0, 1)$; and (ii) if $P^*(t''; \delta) < P^m(t'')$, then $P^*(t'; \delta) > P^*(t''; \delta)\ \forall \delta \in [(n-1)/n, \hat{\delta})$.

Proof. See the Appendix.

Theorem 7 is a direct implication of the fact that optimal collusive profits weakly lead the business cycle and decline throughout a recession (see Theorem 6). This implies that if the market demand function in periods t' and t'' is the same, and if t' is a boom period and t'' is a recessionary period, then profits must be at least as great at t' as at t'', which means that price must be at least as great at t' as at t''. Intuitively, if the demand functions at two points of the cycle are the same, then the gain to cheating will be the same for any given price. However, if demand is rising at one point and falling at the other point, the forgone collusive profits from cheating are lower for the latter. To offset the weaker punishment for cheating, firms must set a lower price when demand is falling.

Theorem 4 showed that it is most difficult to sustain the joint profit-maximizing price when demand is falling. Theorem 7 showed that price is lower when demand is falling than when demand is rising, *ceteris paribus.* These two results support the hypothesis that collusion is more difficult during a recession than during a boom. This is to be contrasted with the finding of Rotemberg and Saloner (1986). Their analysis revealed that firms find it most difficult to collude when the level of market demand is high. In contrast to our model, Rotemberg and Saloner assumed that demand is subject to (observable) i.i.d. shocks, an important implication of which is that future demand is unrelated to current demand. As a result, the discounted loss from cheating is independent of the level of current demand. Given that the gain from cheating is strictly increasing in the level of current demand, Rotemberg and Saloner found that it is more difficult for firms to collude when demand is strong. Our analysis reveals that this finding is quite sensitive to the assumption that future demand is unrelated to current demand. An important factor influencing the incentive to cheat is the projected future path of demand. During a recession, the demand function is shifting in, so the forgone collusive profits over the near future from cheating are anticipated by firms to be relatively low. That is, firms expect collusive profits to be low in the near future, so the cost of inducing a price war is relatively low. By the same logic, when demand is shifting out during a boom, firms anticipate losing large collusive profits if they undercut the collusive price. As a result, the cost to cheating is greater when demand is rising, while the gain to cheating is greater the higher the current demand. Thus, fixing the current demand function, there is a greater incentive to deviate during a recession. We expect that the incentive to cheat would be particularly strong during the early stages of a recession as demand is still relatively high, so that the gain to cheating is great, and as demand is falling, so that the cost to cheating is relatively low. The implication of this incentive for pricing behavior is that prices must be set lower during recessions than during booms.

In summary, the analysis of Rotemberg and Saloner (1986) explores the relationship between the *level* of demand and the difficulty of collusion, while the analysis of this article explores the relationship between the *level* and *direction* of demand and the difficulty of collusion. Once allowing for this second factor, firms find it most difficult to collude when demand is declining, and this results in systematically lower prices during recessions than during booms.

4. Numerical simulations

■ Using the optimal collusive pricing strategy identified in Theorem 1, we parameterized the model and conducted numerical simulations, which are presented in this section. Specifically, we assumed $D(P; t) = \alpha_t - 400P_t$ and $c = 0$. An eight-point cycle is specified in which the demand intercept α_t varies over the cycle. Two different patterns of demand are examined, as noted in the figures.[11] We consider variations in the discount factor over the range $[(n-1)/n, \hat{\delta}]$ and also variations in n for a given discount factor.[12]

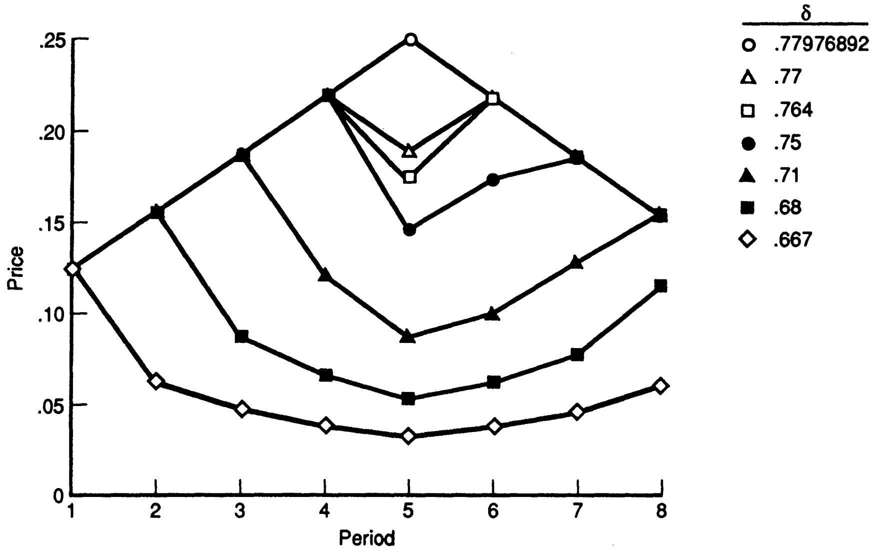
Figures 1 and 2 report the behavior of prices and profits for the case of $n = 3$ and Pattern 1 demand. For this demand pattern, the toughest point to collude is the peak. For

[11] We have considered numerous alternative patterns of demand, and the results are very similar to those reported here. For particular symmetric demand patterns (as in Figures 1 and 2), the toughest point to collude is the peak. In contrast, for demand patterns that involve sufficiently rapid rates of change of demand in the midst of a recession, the toughest point will be other than the peak. Note in this regard that it is possible for the toughest point to collude to be other than the peak, even if the demand pattern is symmetric (e.g., if $\alpha_t = \{100, 101, 102, 199, 200, 199, 102, 101\}$ and $n = 3$, then the toughest point to collude is period 6).

[12] The range of critical discount factors examined in the figures is for the most part quite low relative to the economically meaningful range. This is primarily the consequence of the use of infinite punishments. We maintain the assumption of infinite punishments in this section in order to maintain consistency with the analytical analysis (and much of the literature). In numerical simulations not reported here, we have investigated the impact of allowing for finite punishments. The qualitative nature of the results is quite similar to those reported here, but the range of critical discount factors falls more readily into the economically meaningful range.

FIGURE 1

CARTEL PRICE BEHAVIOR OVER THE CYCLE
Pattern 1* $n = 3$

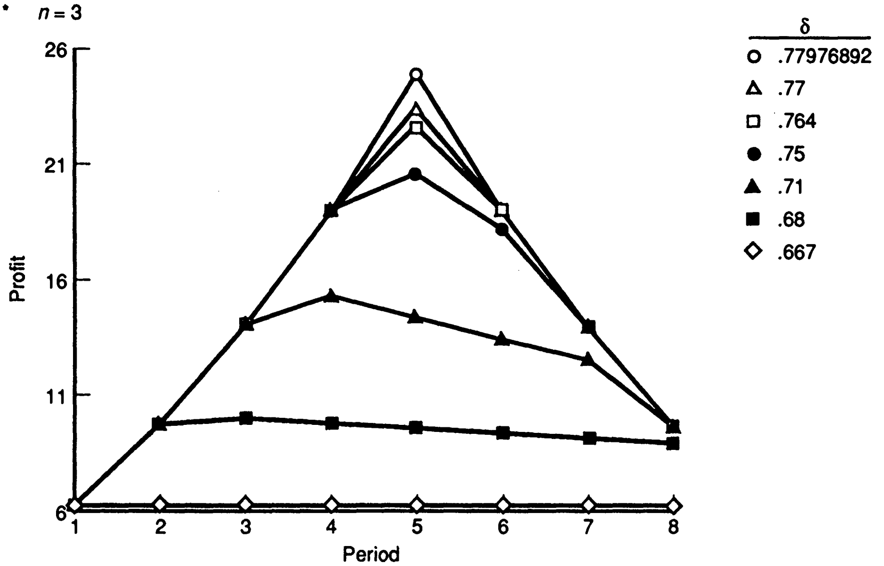


* Pattern 1 demand is $\alpha_t = \{100, 125, 150, 175, 200, 175, 150, 125\}$.

$\delta \geq \hat{\delta}$, Figures 1 and 2 indicate that both prices and profits are strongly procyclical, in accordance with the pattern of monopoly prices and profits. However, as δ falls just below $\hat{\delta}$, it is the price at the peak that must be reduced in order to sustain cooperation. As δ falls further, prices for other periods must be reduced as well to sustain cooperation. Profits remain single-peaked, with the peak occurring during the boom, but they become increasingly

FIGURE 2

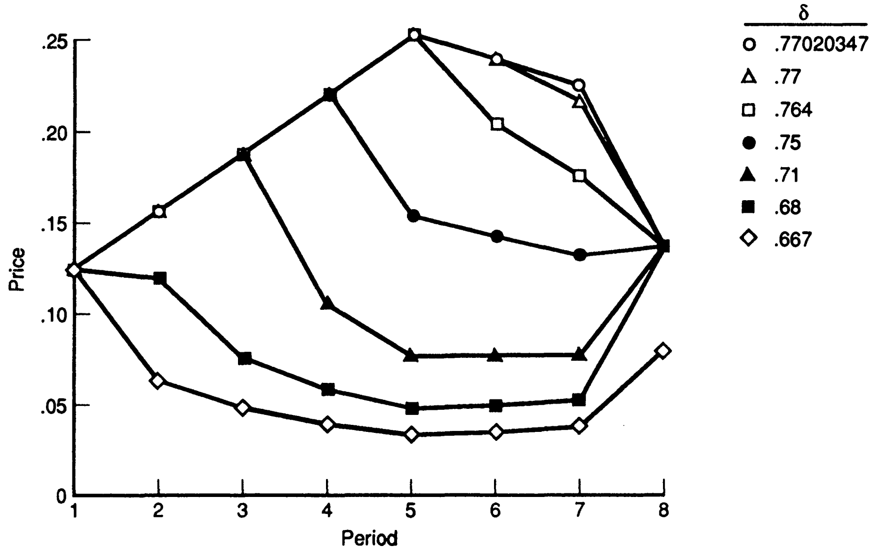
CARTEL PROFIT BEHAVIOR OVER THE CYCLE
Pattern 1* $n = 3$



* Pattern 1 demand is $\alpha_t = \{100, 125, 150, 175, 200, 175, 150, 125\}$.

FIGURE 3

CARTEL PRICE BEHAVIOR OVER THE CYCLE
Pattern 2* $n = 3$



* Pattern 2 demand is $\alpha_t = \{100, 125, 150, 175, 200, 190, 180, 110\}$.

flatter. Observe that it is the prices during the downturn that are reduced prior to the prices during the upturn. This asymmetry in the pattern of price reduction, even though demand is symmetric, illustrates the implications of Theorem 7. Observe further that for any discount factor, the number of periods in which price is increasing in the level of demand is always greater for booms than for recessions. This asymmetry reveals a general tendency for pricing patterns to be procyclical during booms and countercyclical during recessions.

Figures 3 and 4 examine the behavior of prices and profits for Pattern 2 demand and $n = 3$. Observe that as δ falls below $\hat{\delta}$, it is period 7 for which prices must be reduced first in order to sustain cooperation (i.e., period 7 is the toughest point to collude). This indicates that price reductions to sustain cooperation (what might be observationally interpreted as price wars) can be quite prevalent during recessions and in fact during several periods after a downturn has occurred. Observe further that for all ranges of δ, price is single-peaked during the boom. In contrast, for δ in the range .71 to .75, observe that price initially falls, then rises, and then falls again during the recession. This reveals a general tendency in this setting that pricing is single-peaked during booms but may be multipeaked during recessions.[13]

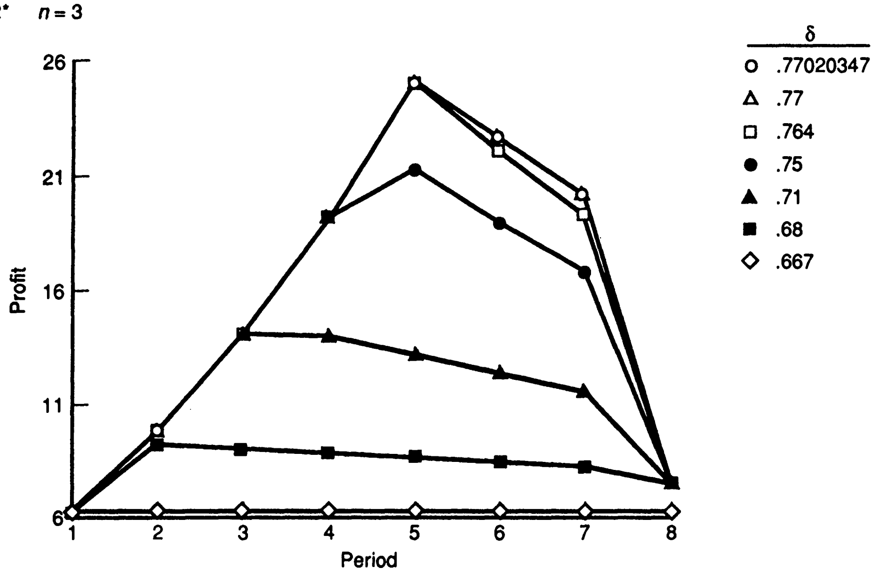
A key parameter in this process is the number of firms. Figures 5 and 6 examine the optimal collusive price and profit patterns as n varies. In these simulations, we use Pattern 2 demand and let $\delta = 6/7$. These simulations reveal that as n becomes sufficiently high, the unconstrained joint profit-maximizing price path is not supportable, and predicted price behavior becomes less procyclical and more asymmetric. That is, over this range price is positively related to the level of demand when demand is rising but negatively related to the level of demand when demand is falling.

Overall, these numerical simulations illustrate two key findings. First, they illustrate

[13] It can be formally demonstrated that prices will be single-peaked during booms (periods of rising demand) but can be multipeaked during recessions (periods of declining demand).

FIGURE 4

CARTEL PROFIT BEHAVIOR OVER THE CYCLE
Pattern 2* $n = 3$

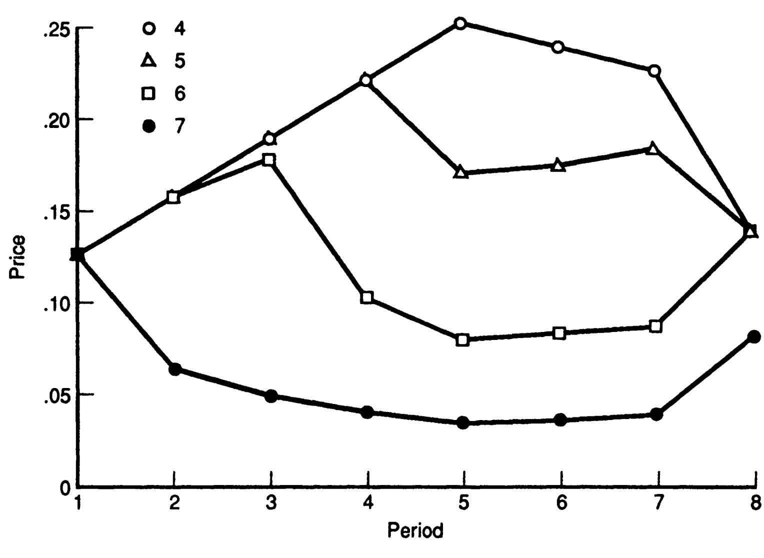


* Pattern 2 demand is α_t = {100, 125, 150, 175, 200, 190, 180, 110}.

our analytical results that the toughest point to collude is during a recession and, accordingly, that price reductions to sustain collusion are more likely during recessions than booms. Second, they illustrate the important resulting implication of a greater tendency for price to be positively related to the level of demand during booms than during recessions.

FIGURE 5

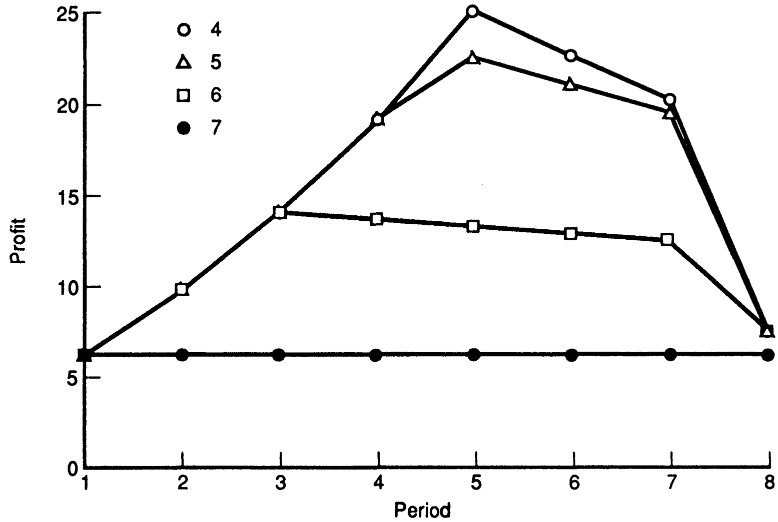
CARTEL PRICE BEHAVIOR OVER THE CYCLE
Pattern 2* $\delta = 6/7$



* Pattern 2 demand is α_t = {100, 125, 150, 175, 200, 190, 180, 110}.

FIGURE 6

CARTEL PROFIT BEHAVIOR OVER THE CYCLE
Pattern 2* δ = 6/7



* Pattern 2 demand is α_t = {100, 125, 150, 175, 200, 190, 180, 110}.

5. Concluding remarks

■ This article has investigated the impact of cyclical movements in market demand on collusive pricing. Two key features to cyclic fluctuations are identified. First, the gain to deviation from the established pricing rule varies over the cycle and is highest when demand is strongest. Second, the discounted loss from such a deviation is also found to vary over the cycle and is lowest during a recession as demand is anticipated to be falling in the immediate future. While the former feature was allowed for in the model of Rotemberg and Saloner (1986), the latter was not. Thus, we found that the toughest time for firms to collude is during a period of falling demand, i.e., a recession, as the forgone profits from deviation are relatively low compared to the current gain.

The finding that the toughest point to collude is during a recession yields interesting predictions regarding collusive pricing behavior over the cycle. First, the current collusive price is not just a function of current demand but also of firm's expectations on future demand. This insight allows us to identify important potential asymmetries in collusive pricing behavior across booms and recessions. In particular, we find that for the same level of demand, the collusive price will always be lower during periods of falling demand than during periods of rising demand. This general asymmetric pattern yields the possibility that price may be procyclical during booms and countercyclical during recessions.

These predictions on pricing behavior are in principle testable and provide new guidance with respect to the specification of empirical studies of cyclical variation in price-cost margins. Existing studies of the latter typically focus on the relationship between a change in price (or the price-cost margin) and a contemporaneous change in demand (proxies that have been used include GNP, capacity utilization, and the aggregate unemployment rate). In addition to current demand, our analysis adds the projected level of *future* demand as a determinant of the intertemporal pricing path. Further, the potential asymmetries in the response of price to the level of demand across booms and slumps that we have identified point to empirical specifications that incorporate and test for such asymmetries.

Appendix

■ Proofs of Theorems 1 through 7 follow.

Proof of Theorem 1. Since $(P - c)D(P; t)$ is bounded in $P\ \forall t$ and $\delta \in (0, 1)$, the payoff function

$$\sum_{t=1}^{\infty} \delta^t[(P(t) - c)(D(P(t); t)/n)]$$

is defined for all $\{P(\tau)\}_{\tau=1}^{\infty}$. Furthermore, it is continuous in $\{P(\tau)\}_{\tau=1}^{\infty}$ by the continuity of the market demand function (Assumption 1). Next we want to show that $\Delta(\delta)$ is a nonempty compact space. First note that $\Delta(\delta)$ is a subset of $\prod_{t=1}^{\infty} [c, \bar{P}(t)]$. Since $\bar{P}(t)$ is bounded $\forall t$, then $[c, \bar{P}(t)]$, being a closed and bounded subset of $\mathbb{R}$, is compact. By Tychonoff's product theorem, $\prod_{t=1}^{\infty} [c, \bar{P}(t)]$ is then compact. Given that both $L(t; \{P(\tau)\}_{\tau=1}^{\infty}, \delta)$ and $G(t; \{P(\tau)\}_{\tau=1}^{\infty})$ are continuous in $\{P(\tau)\}_{\tau=1}^{\infty}$, one can show that $\Delta(\delta)$ is a closed set. Since $\Delta(\delta)$ is a closed subset of a compact space, then $\Delta(\delta)$ is compact. Finally, $\Delta(\delta)$ is nonempty as $\{c\}_{\tau=1}^{\infty} \in \Delta(\delta)$. Since $\{P^*(t; \delta)\}_{t=1}^{\infty}$ is then the maximum of a continuous function over a nonempty compact space, $\{P^*(t; \delta)\}_{t=1}^{\infty}$ exists $\forall \delta \in (0, 1)$. *Q.E.D.*

Proof of Theorem 2. Letting $\pi^m(t) \equiv (P^m(t) - c)D(P^m(t); t)$, the subgame perfect equilibrium conditions (as expressed in (3)) take the following form when the collusive price path is the joint profit maximum.

$$\begin{aligned} L(t; \delta) \equiv L(t; \{P^m(\tau)\}_{\tau=1}^{\infty}, \delta) &= (1/(1 - \delta^{\bar{t}}))[\delta(\pi^m(t + 1)/n) + \delta^2(\pi^m(t + 2)/n) + \ldots \\ &\quad + \delta^{\bar{t}-t}(\pi^m(\bar{t})/n) + \delta^{\bar{t}-t+1}(\pi^m(1)/n) + \ldots + \delta^{\bar{t}}(\pi^m(t)/n)] \\ &\geq (n - 1)(\pi^m(t)/n) = G(t; \{P^m(\tau)\}_{\tau=1}^{\infty}) \equiv G(t),\ t \in \{1, \ldots, \bar{t}\}. \end{aligned} \tag{A1}$$

First note that $L(t; 0) = 0 < G(t)$, $\lim_{\delta\to1} L(t; \delta) = +\infty > G(t)$, and $\partial L(t; \delta)/\partial\delta > 0 = \partial G(t)/\partial\delta$. By the continuity of $L(t; \delta)$ in δ, there exists $\hat{\delta}(t) \in (0, 1)$ such that $L(t; \delta) \geq G(t)$ iff $\delta \geq \hat{\delta}(t)$. Hence, the price path $\{P^m(t)\}_{t=1}^{\infty}$ is a subgame perfect equilibrium outcome iff $\delta \geq \hat{\delta} = \max\{\hat{\delta}(1), \ldots, \hat{\delta}(\bar{t})\}$.

Since $\hat{\delta}(t) < 1\ \forall t \in \{1, \ldots, \bar{t}\}$ then $\hat{\delta} < 1$. To complete the proof of Theorem 2, we have only to show that $\hat{\delta} > ((n - 1)/n)$. Since $\pi^m(\hat{t}) > \pi^m(t)\ \forall t \in \{(1, \ldots, \bar{t}\} - \{\hat{t}\}\}$, then $(\delta/(1 - \delta))[\pi^m(\hat{t})/n] > L(\hat{t}; \delta)$. Given that $(\delta/(1 - \delta))[\pi^m(\hat{t})/n] = (n - 1)[\pi^m(\hat{t})/n]$ for $\delta = ((n - 1)/n)$, then by (A1), $L(\hat{t}; (n - 1)/n)) < (n - 1)[\pi^m(\hat{t})/n]$. Therefore, $\hat{\delta}(\hat{t}) > ((n - 1)/n)$, which implies $\hat{\delta} > ((n - 1)/n)$. *Q.E.D.*

Proof of Theorem 3. Define $\pi^*(t) \equiv (P^*(t; \delta) - c)D(P^*(t; \delta); t)$. We know there exists $t^0 \in \{1, \ldots, \bar{t}\}$ such that $\pi^*(t^0) \geq \pi^*(t)\ \forall t \in \{1, \ldots, \bar{t}\}$. By the conditions for subgame perfection in (3) and the definition of $\{P^*(\tau)\}_{\tau=1}^{\infty}$, it follows that

$$L(t^0; \{P^*(\tau)\}_{\tau=1}^{\infty}, \delta) \geq ((n - 1)/n)\pi^*(t^0). \tag{A2}$$

Since $\pi^*(t^0) \geq \pi^*(t)\ \forall t$, then $(\delta/(1 - \delta))(\pi^*(t^0)/n)$ exceeds the left-hand side of (A2). Therefore, a necessary condition for (A2) to be true is that $(\delta/(1 - \delta))(\pi^*(t^0)/n) \geq ((n - 1)/n)\pi^*(t^0)$, which is equivalent to $\delta \geq ((n - 1)/n)$ when $\pi^*(t^0) > 0$. Hence, if $\delta < ((n - 1)/n)$, then (A2) holds if and only if $\pi^*(t^0) = 0$. Since $\pi^*(t^0) \geq \pi^*(t)\ \forall t$ then $\pi^*(t^0) = 0$ implies that $\pi^*(t) = 0\ \forall t$ so that $P^*(t) = c\ \forall t$. *Q.E.D.*

Proof of Theorem 4. It is straightforward to show that there exists $\tilde{\delta} \in (((n - 1)/n), \hat{\delta})$ such that there exists $t^* \in \{1, \ldots, \bar{t}\}$ such that $P^*(t^*) < P^m(t^*)$ and $P^*(t) = P^m(t)\ \forall t \in \{\{1, \ldots, \bar{t}\} - \{t^*\}\}\ \forall \delta \in [\tilde{\delta}, \hat{\delta})$ and $P^*(t^*) < P^m(t^*)\ \forall \delta \in (0, \hat{\delta})$. Furthermore, t^* is defined by $\hat{\delta} = \hat{\delta}(t^*)$. (For details, see Haltiwanger and Harrington (1987).) To prove Theorem 4, we then need only show that $\hat{\delta} > \hat{\delta}(t)\ \forall t \in \{1, \ldots, \hat{t} - 1\}$, from which it follows that $t^* \notin \{1, \ldots, \hat{t} - 1\}$. Since t^* exists, then t^* must lie in $\{\hat{t}, \ldots, \bar{t}\}$.

To begin, define $f(t)$ as follows:

$$f(t) \equiv \max\{\tau \mid \pi^m(\tau) \geq \pi^m(t),\ \tau \in \{t + 1, \ldots, \bar{t}\}\},\ t \in \{1, \ldots, \hat{t}\}. \tag{A3}$$

$f(t)$ is the maximum point of the cycle at which monopoly profits are at least as great as monopoly profits at point t. It is clear that $f(t) \in \{\hat{t}, \ldots, \bar{t}\}$ since $\pi^m(\hat{t}) > \pi^m(t)$ if $t \in \{1, \ldots, \hat{t} - 1\}$. The method of our proof will be to show that

$$L(t; \delta) - G(t) > L(f(t); \delta) - G(f(t))\ \forall t \in \{1, \ldots, \hat{t} - 1\}. \tag{A4}$$

Since the condition in (A4) implies that $\hat{\delta}(t) < \hat{\delta}(f(t))$, this will be sufficient to prove Theorem 4. Let $t' \in \{1, \ldots, \hat{t} - 1\}$ and $t'' = f(t')$.

$$\{L(t';\delta) - G(t')\} - \{L(t'';\delta) - G(t'')\} = \{(1/(1-\delta^{\bar{t}}))[\delta(\pi^m(t'+1)/n) + \ldots + \delta^{\bar{t}-t'}(\pi^m(\bar{t})/n) + \ldots + \delta^{\bar{t}}(\pi^m(t')/n)] - (n-1)(\pi^m(t')/n)\} - \{(1/(1-\delta^{\bar{t}}))[\delta\pi^m(t''+1) + \ldots + \delta^{\bar{t}-t''}(\pi^m(\bar{t})/n) + \ldots + \delta^{\bar{t}}(\pi^m(t'')/n)] - (n-1)(\pi^m(t'')/n)\}. \quad (A5)$$

By the definition of $f(t)$, we know that $\pi^m(t'') \geq \pi^m(t')$. Therefore, $(n-1)(\pi^m(t'')/n) \geq (n-1)(\pi^m(t')/n)$. Thus, the expression in (A5) is positive if

$$\delta\pi^m(t'+1) + \ldots + \delta^{\bar{t}}\pi^m(t') > \delta\pi^m(t''+1) + \ldots + \delta^{\bar{t}}\pi^m(t''). \quad (A6)$$

Define:

$$A \equiv \delta\pi^m(t'+1) + \ldots + \delta^{t''-t'}\pi^m(t''), \quad (A7)$$

$$B \equiv \delta\pi^m(t''+1) + \ldots + \delta^{\bar{t}-t''+t'}\pi^m(t'). \quad (A8)$$

By these definitions, the condition in (A6) is equivalent to

$$A + \delta^{t''-t'}B > B + \delta^{\bar{t}-t''+t'}A \rightarrow \quad (A9)$$

$$[A/(1-\delta^{t''-t'})] > [B/(1-\delta^{\bar{t}-t''+t'})]. \quad (A10)$$

The expression on the left-hand side of (A10) is the present value of the stream $(\pi^m(t'+1), \ldots, \pi^m(t''))$ received every $t'' - t'$ periods. The right-hand side is the present value of the stream

$$(\pi^m(t''+1), \ldots, \pi^m(\bar{t}), \ldots, \pi^m(t'+1))$$

received every $\bar{t} - t'' + t'$ periods. If $t'' = f(t')$, it is then true that $\pi^m(\tau') > \pi^m(\tau'')\ \forall\tau' \in \{t'+1, \ldots, t''\}$, $\tau'' \in \{t''+1, \ldots, \bar{t}, \ldots, t'+1\}$. The condition in (A10) is then true. This proves that $[L(t;\delta) - G(t)] > [L(f(t);\delta) - G(f(t))]\ \forall t \in \{1, \ldots, \bar{t}-1\}$. Therefore, $\hat{\delta}(t) < \hat{\delta}(f(t))\ \forall t \in \{1, \ldots, \bar{t}-1\}$. We can conclude that $\max\{\hat{\delta}(1), \ldots, \hat{\delta}(\bar{t})\} > \hat{\delta}(t)\ \forall t \in \{1, \ldots, \bar{t}-1\}$. *Q.E.D.*

Proof of Theorem 5. First define the function $V(t; \pi(1), \ldots, \pi(\bar{t}), \delta)$ as follows:

$$V(t;\pi(1), \ldots, \pi(\bar{t}), \delta) \equiv [1/(1-\delta^{\bar{t}})][\delta(\pi(t+1)/n) + \ldots + \delta^{\bar{t}-t}(\pi(\bar{t})/n) + \delta^{\bar{t}-t+1}(\pi(1)/n) + \ldots + \delta^{\bar{t}}(\pi(t)/n)] - (n-1)(\pi(t)/n). \quad (A11)$$

By Theorem 1 we know that $\{P^*(t;\delta)\}_{t=1}^{\infty}$ is defined $\forall\delta \in (0,1)$. By the definition of $P^*(t;\delta)$ and (A11), it is then true that $V(t;\pi^*(1), \ldots, \pi^*(\bar{t}), \delta) \geq 0\ \forall t$. We will show that if one supposes that Theorem 5 is not true, it is then implied that $V(t;\pi^*(1), \ldots, \pi^*(\bar{t}), \delta) < 0$ as $\delta \rightarrow ((n-1)/n)$ for some t. This contradiction will allow us to conclude that Theorem 5 is indeed true.

As a preliminary result, let us show that $P^*(t;\delta) \leq P^m(t)\ \forall t$. Suppose this is not true, so that $P^*(t';\delta) > P^m(t')$ for some $t' \in \{1, 2, \ldots\}$. By reducing price at t' from $P^*(t';\delta)$ to $P^m(t')$, firms raise their profits at t' while keeping cheating unprofitable $\forall t$, which means that their total payoffs go up—which contradicts $\{P^*(t)\}_{t=1}^{\infty}$ being optimal. Cheating is less profitable at t' because the gain to cheating is lower when the collusive price is $P^m(t')$ while the forgone loss is the same. Cheating is also less profitable at $t < t'$ because the forgone loss from cheating is higher, since collusive profits are higher at t'. Finally, for $t > t'$, the gain and loss from cheating are unaffected by changing price at t'.

If Theorem 5 is not true, then $P^*(t;\delta)$ is not countercyclical as $\delta \rightarrow ((n-1)/n)$. In which case, given $P^*(t;\delta) \leq P^m(t)\ \forall t$ and Assumption 5, $\pi^*(t)$ cannot be constant across t as $\delta \rightarrow ((n-1)/n)$. Hence, there exists t' such that $\pi^*(t') \geq \pi^*(t)\ \forall t$ and $\pi^*(t') > \pi(t)$ for some $t \neq t'$ as $\delta \rightarrow ((n-1)/n)$. Since

$$V(t;\pi(1), \ldots, \pi(t), \delta)$$

is increasing in $\pi(\tau)\ \forall\tau \neq t$, then

$$V(t';\pi^*(1), \ldots, \pi^*(\bar{t}), \delta) < V(t';\pi^*(t'), \ldots, \pi^*(t'), \delta) \text{ as } \delta \underset{+}{\rightarrow} ((n-1)/n). \quad (A12)$$

The right-hand expression in (A12) is $V(t';\cdot,\delta)$ evaluated when profits are constant at $\pi^*(t')$. Given that $\lim_{\delta\rightarrow}((n-1)/n)V(t';\pi^*(t'), \ldots, \pi^*(t'), \delta) = 0$, then by (A12), $V(t';\pi^*(1), \ldots, \pi^*(\bar{t}), \delta) < 0$ as $\delta \rightarrow ((n-1)/n)$. Therefore, (A11) does not hold for $t = t'$ as $\delta \rightarrow ((n-1)/n)$. Since a contradiction has been found while presuming that Theorem 5 is false, we conclude that Theorem 5 is indeed true. *Q.E.D.*

Proof of Theorem 6. As a preliminary result, we need to show that if optimal profits are less than monopoly profits at t', then optimal profits at t' exceed optimal profits at $t'+1$.

Lemma 1. $\forall \delta \in ((n-1)/n, 1)$, if $\pi^*(t') < (\pi^m(t')/n)$, then $\pi^*(t') > \pi^*(t'+1)$.

Proof. Let $G^*(t)$ denote the equilibrium gain and $L^*(t; \delta)$ the equilibrium discounted loss from deviating at point t. Let us first show that if $\pi^*(t') < \pi^m(t')$, the $L^*(t'; \delta) = G^*(t')$, so that the incentive compatibility constraint is binding. Suppose not, so that $L^*(t'; \delta) > G^*(t')$. Define $\{P^{**}(t; \delta)\}_{t=1}^{\infty}$ by $P^{**}(t; \delta) = P^*(t; \delta)$ $\forall t \neq t'$ and $P^{**}(t'; \delta) = P^*(t'; \delta) + \epsilon$, where $\epsilon > 0$. Since $P^*(t'; \delta) < P^m(t')$ (as $\pi^*(t') < \pi^m(t')$) and the incentive compatibility constraint is not binding, we can then choose ϵ sufficiently small so that

$$L(t'; \{P^{**}(\tau; \delta)\}_{\tau=1}^{\infty}, \delta) \geq G^*(t'; \{P^{**}(\tau; \delta)\}_{\tau=1}^{\infty}).$$

Thus, cheating is still unprofitable at t' and, furthermore, cheating is still unprofitable at $t \neq t'$ since the forgone loss from cheating is at least as high as before, given that collusive profits are now higher at t'. Therefore, $\{P^{**}(\tau; \delta)\}_{\tau=1}^{\infty} \in \Delta(\delta)$, and it yields a higher payoff for firms (since profits are higher at t' and the same everywhere else), which contradicts the optimality of $\{P^*(t; \delta)\}_{t=1}^{\infty}$. We then conclude that if $\pi^*(t') < \pi^m(t')$, then $L^*(t'; \delta) = G^*(t')$.

Recall that $G^*(t) = (n-1)(\pi^*(t)/n)$, so that $\pi^*(t') > \pi^*(t'+1)$ iff $G^*(t') > G^*(t'+1)$. We will prove that $G^*(t') > G^*(t'+1)$. Given that $L^*(t'; \delta) = G^*(t')$, then $G^*(t') > G^*(t'+1)$ iff $L^*(t'; \delta) > G^*(t'+1)$. Since $L^*(t; \delta) = \delta(\pi^*(t+1)/n) + \delta L^*(t+1; \delta)$, then $L^*(t'; \delta) > G^*(t'+1)$ iff

$$\delta(\pi^*(t'+1)/n) + \delta L^*(t'+1; \delta) > (n-1)(\pi^*(t'+1)/n). \tag{A13}$$

Solving (A13) for $L^*(t'+1; \delta)$, one gets

$$L^*(t'+1; \delta) > [((n-1)/\delta) - 1](\pi^*(t'+1)/n). \tag{A14}$$

Given that $L^*(t'+1; \delta) \geq G^*(t'+1)$ $(=(n-1)(\pi^*(t'+1)/n))$, a sufficient condition for (A14) to be true is that

$$(n-1)(\pi^*(t'+1)/n) > [((n-1)/\delta) - 1](\pi^*(t'+1)/n). \tag{A15}$$

It is straightforward to show that (A15) holds iff $\delta > ((n-1)/n)$, which is true by assumption. We conclude that $G^*(t') > G^*(t'+1)$ and thus $\pi^*(t') > \pi^*(t'+1)$. *Q.E.D.*

With Lemma 1, we can now prove Theorem 6. Let use first show that $\pi^*(\hat{t}) > \ldots > \pi^*(\bar{t}) > \pi^*(1)$. Suppose $\pi^*(t') = (\pi^m(t')/n)$, where $t' \in \{\hat{t}, \ldots, \bar{t}\}$. Since $\pi^*(t') = (\pi^m(t')/n) > (\pi^m(t'+1)/n) \geq \pi^*(t'+1)$, then $\pi^*(t') > \pi^*(t'+1)$. Now suppose $\pi^*(t') < (\pi^m(t')/n)$. By Lemma 1, $\pi^*(t') > \pi^*(t'+1)$. It follows from these two steps that $\pi^*(t) > \pi^*(t+1)$ $\forall t \in \{\hat{t}, \ldots, \bar{t}\}$. Therefore, $\pi^*(\hat{t}) > \ldots > \pi^*(\bar{t}) > \pi^*(1)$.

For the remainder of the proof, let t^0 denote the first point during a boom such that optimal profits are below monopoly profits; that is, $\pi^*(t) = \pi^m(t)$ $\forall t \in \{1, \ldots, t^0 - 1\}$ and $\pi^*(t^0) < \pi^m(t^0)$. Suppose t^0 does not exist so that $\pi^*(t) = \pi^m(t)$ $\forall t \in \{1, \ldots, \hat{t}\}$. Since $\pi^m(1) < \ldots < \pi^m(\hat{t})$, then $\pi^*(1) < \ldots < \pi^*(\hat{t})$. Given we already know that $\pi^*(\hat{t}) > \ldots > \pi^*(\bar{t}) > \pi^*(1)$, it immediately follows that $\pi^*(1) < \ldots < \pi^*(\hat{t}) > \ldots > \pi^*(1)$. In that case, $\tilde{t} = \hat{t}$.

Now suppose $t^0 \in \{1, \ldots, \hat{t}\}$ so that $\pi^*(1) < \ldots < \pi^*(t^0 - 1)$. Since $\pi^*(t^0) < \pi^m(t^0)$, then by Lemma 1, $\pi^*(t^0) > \pi^*(t^0 + 1)$. Let us show that $\pi^*(t) > \pi^*(t+1)$ $\forall t \in \{t^0 + 1, \ldots, \hat{t}\}$. Given that $\pi^m(t^0 + 1) > \pi^m(t^0) > \pi^*(t^0) > \pi^*(t^0 + 1)$, then $\pi^m(t^0 + 1) > \pi^*(t^0 + 1)$, from which it follows once again from Lemma 1 that $\pi^*(t^0 + 1) > \pi^*(t^0 + 2)$. Continuing in this manner, we can infer that $\pi^*(t) > \pi^*(t+1)$ $\forall t \in \{t^0 + 1, \ldots, \hat{t}\}$. Thus far we know that $\pi^*(1) < \ldots < \pi^*(t^0 - 1)$ and $\pi^*(t^0) > \ldots > \pi^*(\hat{t}) > \ldots > \pi^*(1)$. If $\pi^*(t^0 - 1) > \pi^*(t^0)$, then $\pi^*(1) < \ldots < \pi^*(t^0 - 1) > \ldots > \pi^*(1)$ so that $\tilde{t} = t^0 - 1$. If instead $\pi^*(t^0 - 1) < \pi^*(t^0)$, then $\pi^*(1) < \ldots < \pi^*(t^0) > \ldots \pi^*(1)$ so that $\tilde{t} = t^0 + 1$.

Finally, let us show that $t^0 \neq 1$. If $t^0 = 1$, it must be true that $\pi^*(1) > \ldots > \pi^*(\bar{t})$. However, this contradicts our earlier result that $\pi^*(^{-}) > \pi^*(1)$. *Q.E.D.*

Proof of Theorem 7. We will use Theorem 6 to prove Theorem 7. Recall that $D(P; t') = D(P; t'')$ $\forall P \geq 0$, where $t' < \hat{t} < t''$. Let us first show that $P^*(t'; \delta) \geq P^*(t''; \delta)$ $\forall \delta \in (0, 1)$. If $\delta \in (0, (n-1)/n)$, then $P^*(t; \delta) = c$ $\forall t$ so that $P^*(t'; \delta) = P^*(t''; \delta)$. If $\delta = ((n-1)/n)$, then $\pi^*(t) = \pi^m(1)$ $\forall t$, which implies $P^*(t'; \delta) = P^*(t''; \delta)$. If $\delta \in [\hat{\delta}, 1]$, then $\pi^*(t) = \pi^m(t)$ $\forall t$ so that $P^*(t'; \delta) = P^m(t') = P^*(t''; \delta)$. Finally, consider the case when $\delta \in ((n-1)/n, \hat{\delta})$. Suppose $\pi^*(t') = \pi^m(t')$ so that $P^*(t'; \delta) = P^m(t')$. Since $P^*(t; \delta) \leq P^m(t)$ $\forall t$, then $P^*(t''; \delta) \leq P^m(t'') = P^m(t') = P^*(t''; \delta)$. Hence, $P^*(t'; \delta) \geq P^*(t''; \delta)$. Now suppose $\pi^*(t') < \pi^m(t')$. It follows from Theorem 6 that $t' \geq \tilde{t}$ and, therefore, $\pi^*(t') > \ldots > \pi^*(\bar{t})$. Hence, $\pi^*(t') > \pi^*(t'')$, which implies $P^*(t'; \delta) > P^*(t''; \delta)$. We have then shown that $P^*(t'; \delta) \geq P^*(t''; \delta)$ $\forall \delta \in (0, 1)$.

To conclude the proof of Theorem 7, we need to show that if $P^*(t''; \delta) < P^m(t'')$, then $P^*(t'; \delta) > P^*(t''; \delta)$. Suppose not; then, since $P^*(t'; \delta) \geq P^*(t''; \delta)$, it follows that $P^*(t'; \delta) = P^*(t''; \delta)$. Given that $P^*(t''; \delta) < P^m(t'')$, then $P^*(t'; \delta) < P^m(t')$, which implies $\pi^*(t') < \pi^m(t')$. However, we have already shown that when $\pi^*(t') < \pi^m(t')$, $P^*(t'; \delta) > P^*(t''; \delta)$: a contradiction. *Q.E.D.*

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