JOURNAL OF ECONOMIC THEORY 22, 327–338 (1980)

Entry (and Exit) in a Differentiated Industry*

J. JASKOLD GABSZEWICZ AND J.-F. THISSE

*Center for Operations Research & Econometrics,
Université Catholique de Louvain, Louvain-la-Neuve 1348, Belgium*

Received August 6, 1979

The entry process in an industry embodying more or less close substitutes is considered. One examines whether the increase in the number of substitutes induces pure competition when prices are chosen noncooperatively. It is shown that there exists an upper bound on the number of firms which can compete in the market: when this upperbound is reached, any further entry entails the exit of an existing firm. In spite of this fact, new entries imply the decrease of prices to the competitive ones.

Since Cournot [3], there has been a long-standing tradition according to which entry into a homogeneous market, where oligopolists use quantity strategies, restores pure competition (see, for instance, [5] or [7]). Intuitively, when the number of firms increases, the ability of each oligopolist to alter the value of the inverse demand function through his own strategic choice must necessarily diminish, and vanishes at the limit. By contrast, if the firms use price strategies on the same market, it has been known since Bertrand [1] that pure competition obtains already with two firms. Clearly the loss induced by undercutting the competitor's price is broadly compensated by capturing the whole demand. With Hotelling [4] and Chamberlin [2], the idea was developed that firms operate through product differentiation in order to avoid price competition "à la Bertrand." Nevertheless, within such a context, the problem remains open whether, by analogy with the homogeneous case, the increase in the number of substitutes in the industry induces pure competition when prices are chosen noncooperatively.

In order to deal with this problem, the approach employed for the homogeneous case suggests starting out with an entry process where the entrants arrive in the industry with products which are more or less close substitutes for the existing ones, and then studying the asymptotic behavior of noncooperative prices when the number of entrants tends to infinity. It is the purpose of this article to show through an example that this procedure

* The major part of this work was done while the first author was visiting Bonn University. Financial support from the Sonderforschungsbereich 21 is gratefully acknowledged.

0022-0531/80/020327-12$02.00/0

cannot be transposed as such to the case of a differentiated industry with price strategies.

Our example suggests that *the number of firms which can coexist in a differentiated industry cannot exceed a finite value* n^* (in our example, the number n^* is determined by a set of parameters which describe our simple economy, inspired by a previous work of the authors [6]). Surprisingly, if more than n^* firms try to remain on the market, they will necessarily jostle each other, and this struggle will provoke the exit of one of them. Still more surprising is that *this upper bound on the number of firms does not preclude that entry reinforces the tendency toward pure competition.*

In fact, the entry process decomposes into three successive phases. The first one corresponds to the situation where the number of firms is such that the whole market is not supplied at the equilibrium prices. The second obtains when the whole market is served, but where room is left for the entry of some additional firms. In the third, and last, phase, the number of potential firms is larger than n^*. It will be shown that in both the first and second phases, new entries entail decreases in prices of the products already sold on the market. As for the third phase, a new entry is now necessarily accompanied by the exit of another firm. But, in spite of the fixed number of firms still allowed on the market, equilibrium prices must necessarily decrease to the competitive ones when the number of entrants increases.

Finally, it must be noted that competition can also be restored with means other than the number of firms, because product differentiation adds a new dimension to the rivalry among firms. Our example suggests that, more than from the number of firms, perfect competition could emerge from the close substitutability among the products, thus confirming the "objection péremptoire" of Bertrand against Cournot.

The authors have recently proposed a model for dealing with a situation of differentiated duopoly [6]. Its extension to an arbitrary number of firms can provide a natural framework for settling an example in which the above questions can be discussed.

Imagine an industry constituted by *n firms*, indexed by k, $k = 1,..., n$; firm k sells, at no cost, product k; all these products are more or less close substitutes for each other. Let $T = [0, 1]$ be the set of *consumers*, which are assumed to be ranked in T by order of increasing income, and let the income $R(t)$ of consumer $t \in T$ be given by

$$R(t) = R_1 + R_2 t, \qquad R_1 > 0,\ R_2 \geqslant 0.$$

Consumers are also assumed to make indivisible and mutually exclusive purchases. Thus if consumer t decides to buy one of the products, k, he buys that product only, and a single unit of it.

Let us denote by $u(k, R)$ the utility of having one unit of product k and an income R, and by $u(0, R)$ the utility of having no unit of any product and an income R. In our example we take the further specification[1]

$$u(k, R) = u_k \cdot R = u_1 \cdot [1 + \alpha(k - 1)] \cdot R, \qquad \alpha \geqslant 0, \tag{1}$$

and

$$u(0, R) = u_0 \cdot R \tag{2}$$

with $u_1 > u_0 > 0$.

In this specification, u_k is a "utility index" which ranks the quality of the products: if $k > h$, product k is more desired than product h. The parameter α is a measure of the substitutability between products k; thus, for $\alpha = 0$ all products are perfect substitutes, and substitutability decreases when α increases.

Let p_k be the price quoted by oligopolist k; product k would be bought by customer t rather than product j, $j \neq k$, if and only if

$$\begin{aligned} u(k, R(t) - p_k) &= u_1 \cdot [1 + \alpha(k - 1)](R_1 + R_2 t - p_k) \\ &\geqslant u_1 \cdot [1 + \alpha(j - 1)](R_1 + R_2 t - p_j) = u(j, R(t) - p_j). \end{aligned} \tag{3}$$

To be sure that t buys product k, we must further have

$$\begin{aligned} u(k, R(t) - p_k) &= u_1 \cdot [1 + \alpha(k - 1)](R_1 + R_2 t - p_k) \\ &\geqslant u_0 \cdot (R_1 + R_2 t) = u(0, R(t)). \end{aligned} \tag{4}$$

(The consumer must prefer to buy product k rather than nothing.)

In what follows we want to characterize a noncooperative price equilibrium for the above framework. By this we mean an n-tuple of prices such that no firm can increase its profit by any unilateral deviation and such that each of the n products obtains a positive market share. To this end, let us derive the contingent demand function for each oligopolist, under the assumption that each of the n firms has a positive market share. Let $M_k(\bar{p}_1, \ldots, p_k, \ldots, \bar{p}_n)$ be the *market share* of firm k defined by

$$M_k(\bar{p}_1, \ldots, p_k, \ldots, \bar{p}_n) = \{t \mid t \text{ buys product } k \text{ at prices } (\bar{p}_1, \ldots, p_k, \ldots, \bar{p}_n)\},$$

and let (t, τ) be a pair of consumers such that $t < \tau$ (which implies that t is poorer than τ). Using (1), it is easily seen that if t chooses to buy product k, then τ will not choose to buy a product $k - j$ of lower quality, with $j \in$

[1] Note that this formulation does not imply any restriction with respect to our analysis in [6], when $n = 2$.

$\{1,..., k-1\}$. Similarly, again using (1), if τ chooses to buy product k, then t will not choose to buy a product $k+j$ of higher quality, with $j \in \{1,..., n-k\}$. It follows that $M_k(\bar{p}_1,..., p_k,..., \bar{p}_n)$ is defined by an interval I_k of [0, 1] and that the intervals I_k are ranked from the left to the right of [0, 1] in an increasing order. Given our hypothesis that each consumer of brand k buys only a single unit of that product, the contingent demand function of oligopolist k is equal to the length of the interval I_k. To determine that length, it is sufficient to identify both extremities of I_k. First, let k be a product such that $1 < k < n$. The lower extremity of I_k is given by the consumer who is just indifferent between buying product k or product $k-1$, since by assumption all the firms are on the market. Similarly the upper extremity is defined by the consumer who is just indifferent between purchasing product k or product $k+1$. According to (3), the lower and upper extremities of I_k are consequently given by

$$t_k(\bar{p}_1,..., p_k,..., \bar{p}_n) \underset{\text{def}}{=} \frac{u_k p_k - u_{k-1} p_{k-1}}{(u_k - u_{k-1}) R_2} - \frac{R_1}{R_2} \tag{5}$$

and

$$t_{k+1}(\bar{p}_1,..., p_k,..., \bar{p}_n) \underset{\text{def}}{=} \frac{u_{k+1} p_{k+1} - u_k p_k}{(u_{k+1} - u_k) R_2} - \frac{R_1}{R_2}, \tag{6}$$

so that the demand function $\mu[M_k(\bar{p}_1,..., p_k,..., \bar{p}_n)]$ is equal to

$$\mu[M_k(\bar{p}_1,..., p_k,..., \bar{p}_n)] = t_{k+1}(\bar{p}_1,..., p_k,..., \bar{p}_n) - t_k(\bar{p}_1,..., \bar{p}_n). \tag{7}$$

Let us now consider the case of product n. Since no brand of higher quality exists, the upper extremity of I_n is provided by the richest customer $t = 1$, so that the demand function of oligopolist n is given by

$$\begin{aligned}\mu[M_n(\bar{p}_1,..., \bar{p}_k,..., p_n)] &= 1 - t_n(\bar{p}_1,..., \bar{p}_k,..., p_n)\\ &= 1 - \frac{u_n p_n - u_{n-1} p_{n-1}}{(u_n - u_{n-1}) R_2} + \frac{R_1}{R_2}.\end{aligned} \tag{8}$$

Finally let us envision the case of product 1. Since no brand of lower quality is offered and if the whole market is not served, the lower extremity of I_1 is provided by the consumer who is just indifferent between buying product 1 or buying nothing, i.e., using (4) with $k = 1$,

$$t_1(p_1,..., \bar{p}_k,..., \bar{p}_n) \underset{\text{def}}{=} \frac{u_1 p_1}{(u_1 - u_0) R_2} - \frac{R_1}{R_2}; \tag{9}$$

the demand function of oligoplist 1 is then given by

$$\mu[M_1(p_1,..., \bar{p}_k,..., \bar{p}_n)] = t_2(p_1,..., \bar{p}_k,..., \bar{p}_n) - t_1(p_1,..., \bar{p}_k,..., \bar{p}_n). \tag{10}$$

On the other hand, if the whole market is served, the lower extremity of I_1 is defined by the poorest customer $t = 0$, so that the demand function of oligopolist 1 is then equal to

$$\mu[M_1(p_1, ..., \bar{p}_k, ..., \bar{p}_n)] = t_2(p_1, ..., \bar{p}_k, ..., \bar{p}_n). \tag{11}$$

Denote by $P_k(\bar{p}_1, ..., p_k, ..., \bar{p}_n) = p_k \cdot \mu[M_k(\bar{p}_1, ..., p_k, ..., \bar{p}_n)]$ the profit function of firm k. A *noncooperative price equilibrium* is defined as an n-tuple of prices $(p_1^*, ..., p_k^*, ..., p_n^*)$ such that no firm k can increase its profit by any unilateral deviation from p_k^* when other firms j stick to prices p_j^*, $j \neq k$, and $\mu[M_k(p_1^*, ..., p_k^*, ..., p_n^*)] > 0$, $\forall\, k = 1, ..., n$.[2]

Let us now determine the equilibrium prices, successively, when the whole market is not served ($\sum_{k=1}^{n} \mu[M_k(p_1^*, ..., p_n^*)] < 1$) and when it is entirely served ($\sum_{k=1}^{n} \mu[M_k(p_1^*, ..., p_n^*)] = 1$). In the first case, the demand functions are described by (7), (8), and (10). Assuming that there exists a noncooperative price equilibrium, the following first-order conditions must be satisfied:

$$\left.\frac{\partial P_1}{\partial p_1}\right|_{(p_1^*, ..., p_n^*)} = \frac{(1 + \alpha)\, u_1 p_2^* - 2u_1 p_1^*}{\alpha u_1 R_2} - \frac{2u_1 p_1^*}{(u_1 - u_0)\, R_2} = 0,$$

$$\left.\frac{\partial P_k}{\partial p_k}\right|_{(p_1^*, ..., p_n^*)} = \frac{(1 + \alpha k)\, u_1 p_{k+1}^* - 2[1 + \alpha(k-1)]\, u_1 p_k^*}{\alpha u_1 R_2} - \frac{2[1 + \alpha(k-1)]\, u_1 p_k^* - [1 + \alpha(k-2)]\, u_1 p_{k-1}^*}{\alpha u_1 R_2} = 0,$$
$$k = 2, ..., n-1,$$

$$\left.\frac{\partial P_n}{\partial p_n}\right|_{(p_1^*, ..., p_n^*)} = 1 - \frac{2[1 + \alpha(n-1)]\, u_1 p_n^* - [1 + \alpha(n-2)]\, u_1 p_{n-1}^*}{\alpha u_1 R_2} + \frac{R_1}{R_2} = 0,$$

[2] The contingent demand functions we have derived above are only valid in the restricted domains of price strategies for which all the n oligopolists obtain a positive market share. Without this assumption the possibility should indeed be recognized that, for any group of products, n-tuples of prices can be found which would cancel the demand for any product in that group. Nevertheless, it can be shown that the equilibrium prices calculated in the restricted domains are in fact equilibrium prices over the whole domains. The reason is that the profit function of each oligopolist is quasi-concave over the whole domain of its strategies, so that a local best reply is also a global one. This subject will be dealt in a forthcoming paper.

which reduce to the system of difference equations:

$$(1+\alpha)(u_1-u_0)\,p_2^* - 2[(u_1-u_0)+\alpha u_1]\,p_1^* = 0,$$
$$(1+\alpha k)\,p_{k+1}^* - 4[1+\alpha(k-1)]\,p_k^* + [1+\alpha(k-2)]\,p_{k-1}^* = 0,$$
$$k = 2,\ldots, n-1,$$
$$-2[1+\alpha(n-1)]\,p_n^* + [1+\alpha(n-2)]\,p_{n-1}^* = -\alpha(R_1+R_2).$$

Using the change of variables defined by

$$x_k = [1+\alpha(k-1)]\,p_k^* \tag{12}$$

the above system can be rewritten as

$$\begin{aligned}(u_1-u_0)\,x_2 - 2[(u_1-u_0)+\alpha u_1]\,x_1 &= 0,\\ x_{k+1} - 4x_k + x_{k-1} &= 0, \qquad k = 2,\ldots, n-1,\\ -2x_n + x_{n-1} &= -\alpha(R_1+R_2),\end{aligned} \tag*{\{1\}}$$

the solution of which is

$$x_k = A_n(2+3^{1/2})^k + B_n(2-3^{1/2})^k$$

with

$$A_n = \frac{\alpha(2-3^{1/2})[3^{1/2}(u_1-u_0)+2\alpha u_1](R_1+R_2)}{\left((3u_1-3u_0+2\cdot 3^{1/2}\alpha u_1)(2+3^{1/2})^{n-1} - (3u_1-3u_0-2\cdot 3^{1/2}\alpha u_1)(2-3^{1/2})^{n-1}\right)}$$

and

$$B_n = \frac{\alpha(2+3^{1/2})[3^{1/2}(u_1-u_0)-2\alpha u_1](R_1+R_2)}{\left((3u_1-3u_0+2\cdot 3^{1/2}\alpha u_1)(2+3^{1/2})^{n-1} - (3u_1-3u_0-2\cdot 3^{1/2}\alpha u_1)(2-3^{1/2})^{n-1}\right)}.$$

Given (12), we finally obtain

$$p_k^* = \frac{1}{[1+\alpha(k-1)]}\,[A_n(2+3^{1/2})^k + B_n(2-3^{1/2})^k]. \tag{13}$$

These prices are only valid when the whole market is not served. In order to guarantee that this condition is satisfied, we must have, in particular, that the equilibrium price p_1^* for oligopolist 1 obtained from (13) is larger than $[(u_1-u_0)/u_1]\,R_1$ (by (4) with $k=1$), i.e.,

$$\frac{R_1}{R_2} < \frac{2u_1\alpha\, 3^{1/2}}{\left(3(u_1-u_0)[(2+3^{1/2})^{n-1}-(2-3^{1/2})^{n-1}] + 2u_1\alpha\, 3^{1/2}[(2+3^{1/2})^{n-1}+(2-3^{1/2})^{n-1}-1]\right)}. \tag{14}$$

In the case where the whole market is served, the demand functions are defined by (7), (8), and (11). Hence an interior noncooperative price equilibrium must verify the first-order conditions

$$\left.\frac{\partial P_1}{\partial p_1}\right|_{(p_1^*,\ldots,p_n^*)} = \frac{(1+\alpha)\,u_1 p_2^* - 2u_1 p_1^*}{\alpha u_1 R_2} - \frac{R_1}{R_2} = 0,$$

$$\left.\frac{\partial P_k}{\partial p_k}\right|_{(p_1^*,\ldots,p_n^*)} = \frac{(1+\alpha k)\,u_1 p_{k+1}^* - 2[1+\alpha(k-1)]\,u_1 p_k^*}{\alpha u_1 R_1} - \frac{2[1+\alpha(k-1)]\,u_1 p_k^* - [1+\alpha(k-2)]\,u_1 p_{k-1}^*}{\alpha u_1 R_2} = 0,$$
$$k = 2,\ldots, n,$$

$$\left.\frac{\partial P_n}{\partial p_n}\right|_{(p_1^*,\ldots,p_n^*)} = 1 - \frac{2[1+\alpha(n-1)]\,u_1 p_n^* - [1+\alpha(n-2)]\,u_1 p_{n-1}^*}{\alpha u_1 R_2} + \frac{R_1}{R_2} = 0,$$

which, for the change of variables (12), are equivalent to the system of difference equations

$$\begin{aligned} x_2 - 2x_1 &= \alpha R_1, \\ x_{k+1} - 4x_k + x_{k-1} &= 0, \qquad k = 2,\ldots, n-1, \\ -2x_n + x_{n-1} &= -\alpha(R_1 + R_2), \end{aligned} \tag{\{2\}}$$

whose solution is given by

$$x_k = A_n'(2 + 3^{1/2})^k + B_n'(2 - 3^{1/2})^k$$

with

$$A_n' = \frac{(3 - 2\cdot 3^{1/2})[\alpha(2 - 3^{1/2})^{n-1} R_1 - \alpha(R_1 + R_2)]}{3[(2 + 3^{1/2})^{n-1} - (2 - 3^{1/2})^{n-1}]}$$

and

$$B_n' = \frac{(3 + 2\cdot 3^{1/2})[\alpha(2 + 3^{1/2})^{n-1} R_1 - \alpha(R_1 + R_2)]}{3[(2 + 3^{1/2})^{n-1} - (2 - 3^{1/2})^{n-1}]}$$

so that p_k^* is defined by

$$p_k^* = \frac{1}{[1 + \alpha(k-1)]}\,[A_n'(2 + 3^{1/2})^k + B_n'(2 - 3^{1/2})^k]. \tag{15}$$

These prices are only valid when the market is entirely served and when each oligopolist gets a positive market share. To this effect, the equilibrium price p_1^* of oligopolist 1 given by (15) must be smaller than or equal to $[(u_1 - u_0)/u_1] R_1$, i.e.,

$$\frac{2u_1\alpha\, 3^{1/2}}{\left(\begin{array}{l}3(u_1 - u_0)[(2 + 3^{1/2})^{n-1} - (2 - 3^{1/2})^{n-1}] \\ \qquad + u_1\alpha\, 3^{1/2}[(2 + 3^{1/2})^{n-1} + (2 - 3^{1/2})^{n-1} - 2]\end{array}\right)} \leqslant \frac{R_1}{R_2} \tag{16}$$

and $\mu[M_1(p_1^*,..., p_k^*,..., p_n^*)]$ must be positive, i.e.,

$$\frac{R_1}{R_2} < \frac{2}{(2 + 3^{1/2})^{n-1} + (2 - 3^{1/2})^{n-1} - 2}. \tag{17}$$

So far, we have characterized equilibrium prices in an industry embodying n firms selling products whose respective utility indices satisfy relationship (1). Equipped with this framework, we may now study the change in prices and market shares when the number of firms increases. To proceed in that direction, we shall assume that new firms always enter the market with higher-quality products, namely, that the $(n + 1)$st firm enters the market with a utility index u_{n+1} equal to $(1 + \alpha n) u_1$.[3]

To begin with, let us assume that the starting number n of firms is such that the whole market is not served. In this case, we know that condition (14) must hold and that equilibrium prices are given by (13). Defining $\bar{n}$ as the largest integer for which condition (14) is verified, it means that $n \leqslant \bar{n}$. If a new firm enters, then either $n + 1$ is still smaller than or equal to $\bar{n}$, or $n + 1$ is greater than $\bar{n}$. In the first alternative, it is easily verified that *the "after-entry" equilibrium prices for existing firms*, still given by (13), *are smaller than the "pre-entry" equilibrium prices*. Consequently, as long as the whole market is not served, equilibrium prices form a decreasing sequence of the number of the new entrants. In the second alternative, the whole market is served. Assuming that an interior noncooperative price equilibrium is observed with $(\bar{n} + 1)$ firms (which is fulfilled if condition (16) holds for $\bar{n} + 1$), the equilibrium prices are then given by (15). Computing $\partial p_k^*/\partial n$ from (15), we obtain the result that, as soon as the whole market is served, equilibrium prices again form a decreasing sequence when the number of entrants increases.

Our major finding is that this entry process cannot allow a continuously increasing number of firms with a positive market share. Indeed, for all the firms already on the market to maintain a positive share, we know that condition (17), among others, must be satisfied. As the right-hand side of (17) is a decreasing function of n, there exists a maximal number, say n^*, of firms for

[3] Although the use of this particular entry process entails some loss of generality, that loss is comparable with the loss of generality which follows from assuming that all firms are identical, a hypothesis usually made in the theory of entry with homogeneous products. Moreover, analogous processes are considered in the recent literature in location theory (see, for instance, [8]).

which (17) can still hold. For $n > n^*$, the converse of (17) must be verified: income disparities, as expressed in our model by the value of R_2, are no longer sufficient to sustain an industry embodying a larger number of firms. In other words, *the income distribution determines endogenously the maximal number of products which defines the industry.* It is our belief that the number n^* can be viewed as a kind of long-run equilibrium number of firms in the sense that, when this number is reached, no room is left for a larger number of products.

Does it mean that no other firm with a higher-quality product can enter when the long-run equilibrium number n^* is reached? The answer is no. Indeed the entry of a new firm can take place, provided, however, that it is accompanied by the exit of another. Assume that a firm selling a product with utility index $u_{n^*+1} = (1 + \alpha n^*)\, u_1$ decides to enter. It then follows from the definition of n^* that at least one other firm must necessarily obtain a null market share at the after-entry equilibrium. In fact, it can be shown that only one firm must exit at the after-entry equilibrium and that it can be only firm 1 (see the proposition given in the Appendix). Consequently, after entry of firm $n^* + 1$, the industry again embodies n^* firms, but now with indices $\{2,..., n^* + 1\}$. More generally, the entry of firm $n^* + m$, with utility index $u_{n^*+m} = [1 + \alpha(n^* + m - 1)]\, u_1$, would similarly lead to an industry profile defined by the firms $\{m + 1,..., n^* + m\}$. Interestingly, *the after-entry equilibrium prices form a sequence decreasing to the competitive prices as the number m of new entrants increases.* Indeed, the equilibrium price of firm k is given by

$$p_k^* = \frac{1}{1 + \alpha(k - 1)} [A'_{n^*}(2 + 3^{1/2})^{k-m} + B'_{n^*}(2 - 3^{1/2})^{k-m}],$$

with $k = m + 1,..., m + n^*$ (again see the proposition of the Appendix). Among other things, this result implies that we may observe low prices in an industry embodying a fixed, and possibly small, number of firms provided m is large enough.

Let us illustrate the whole process we have just described when the long-run equilibrium number n^* is equal to 2. Let firm 1 initially be a monopolist in the industry and sell a product with utility index u_1. By the choice rule (4) with $k = 1$, one easily checks that the market share $M_1(p_1)$ is defined by the interval I_1 whose lower and upper extremities are respectively given by

$$t_1(p_1) = \text{Max} \left\{0, \frac{u_1 p_1}{(u_1 - u_0)\, R_2} - \frac{R_1}{R_2}\right\}$$

and 1. The demand function $\mu[M_1(p_1)]$ faced by firm 1 is then

$$\text{Min} \left\{1, 1 - \frac{u_1 p_1}{(u_1 - u_0)\, R_2} + \frac{R_1}{R_2}\right\}.$$

Assuming $R_1/R_2 < 1$, a simple calculation shows that the monopoly price is equal to $\frac{1}{2}[(u_1 - u_0)/u_1](R_1 + R_2)$. Since, at this price, all the consumers are not served, room is left for entry of firm 2 with utility index $u_2 = (1 + \alpha) u_1$. Assuming also that condition (16) holds for $n = 2$, namely,

$$\frac{\alpha}{3(u_1 - u_0) + \alpha u_1} \leqslant \frac{R_1}{R_2},$$

we deduce from the above that the whole market is supplied at the equilibrium prices

$$p_1^* = \frac{\alpha(R_2 - R_1)}{3} \quad \text{and} \quad p_2^* = \frac{\alpha(2R_2 + R_1)}{3(1 + \alpha)};$$

consequently, $\bar{n} = 1$.[4] Suppose further that condition (17) is not fulfilled for $n = 3$, namely, that $\frac{1}{6} \leqslant R_1/R_2$; accordingly, $n^* = 2$. Hence under the hypothesis

$$\max \left\{\frac{\alpha}{3(u_1 - u_0) + \alpha u_1}, \frac{1}{6}\right\} \leqslant \frac{R_1}{R_2} < 1,$$

two firms, and only two firms, may remain in the industry forever. If firm 3 with utility index $u_3 = (1 + 2\alpha) u_1$ would in turn enter the market, then a new equilibrium would emerge at which firm 1 has been enforced to exit, with prices

$$p_2^* = \frac{\alpha(R_2 - R_1)}{3(1 + \alpha)} \quad \text{and} \quad p_3^* = \frac{\alpha(2R_2 + R_1)}{3(1 + 2\alpha)}.$$[5]

It remains to study the role of substitutability among the products at the equilibrium. First note that, whatever the fixed number of products, a value of α sufficiently small exists for condition (16) to be satisfied. In this case, equilibrium prices are then given by (15). Second, these equilibrium prices form a decreasing sequence which converges to zero as the substitutability between products increases, that is, as α tends to zero. As stated above, pure competition is the limit of a process where products become more and more homogeneous. At the limit this is nothing else but the "objection péremptoire" of Bertrand against Cournot.

APPENDIX

PROPOSITION. *Let q and m be two arbitrary integers such that $q \leqslant m + 1$ and let the firms defined by the set of indices $\{q,..., n^*,..., n^* + m\}$. A non-*

[4] A more extensive discussion of the market solution for $n = 2$ is contained in [6].

[5] This illustration shows that the long-run equilibrium number is not necessarily very large. A priori, to any value of n, there corresponds an income distribution which authorizes a number of firms at most equal to this value. Possibly, with a high degree of income dispersion, a large number of products will be observed.

cooperative price equilibrium involves exactly n^* firms given by the set of indices $\{m+1, \ldots, n^*+m\}$. Furthermore, the equilibrium price of firm k, with $k = m+1, \ldots, n^*+m$ is given by

$$p_k^* = \frac{1}{1+\alpha(k-1)} \cdot [A'_{n^*}(2+3^{1/2})^{k-m} + B'_{n^*}(2-3^{1/2})^{k-m}]. \quad (18)$$

Proof. Assume that the firms defined by $\{q, \ldots, n^*+m\}$ have a positive market share at the equilibrium and that the firms defined by $\{1, \ldots, q-1\}$ have been put out of business. In this case, the equilibrium prices must verify the following first-order conditions:

$$(1+\alpha q)\, p_{q+1}^* - 2[1+\alpha(q-1)]\, p_q^* - \alpha R_1 = 0,$$

$$(1+\alpha k)\, p_{k+1}^* - 4[1+\alpha(k-1)]\, p_k^* + [1+\alpha(k-2)]\, p_{k-1}^* = 0,$$
$$k = q+1 \cdots n^*+m-1,$$

$$\alpha(R_1+R_2) - 2[1+\alpha(n^*+m-1)]\, p_{n^*+m}^* + [1+\alpha(n^*+m-2)]\, p_{n^*+m-1}^* = 0.$$

Using the change of variables defined by (12), we obtain

$$\begin{aligned} x_{q+1} - 2x_q - \alpha R_1 &= 0, \\ x_{k+1} - 4x_k + x_{k-1} = 0, \qquad k &= q+1, \ldots, n^*+m-1, \\ \alpha(R_1+R_2) - 2x_{n^*+m} + x_{n^*+m-1} &= 0. \end{aligned} \quad \{3\}$$

Furthermore, setting $j = k - q + 1$ yields the system

$$\begin{aligned} x_2 - 2x_1 - \alpha R_1 &= 0, \\ x_{j+1} - 4x_j + x_{j-1} = 0, \qquad j &= 2, \ldots, n^*+m-q, \\ \alpha(R_1+R_2) - 2x_{n^*+m-q+1} + x_{n^*+m-q} &= 0. \end{aligned} \quad \{4\}$$

This system is identical to system {2}, with $n = n^*+m-q+1$, so that its solution is given by

$$x_j = A'_{n^*+m-q+1}(2+3^{1/2})^j + B'_{n^*+m-q+1}(2-3^{1/2})^j,$$
$$j = 1, \ldots, n^*+m-q+1,$$

i.e.,

$$x_k = A'_{n^*+m-q+1}(2+3^{1/2})^{k-q+1} + B'_{n^*+m-q+1}(2-3^{1/2})^{k-q+1},$$
$$k = q, \ldots, n^*+m,$$

which leads to

$$p_k^* = \frac{1}{1 + \alpha(k-1)} \cdot [A'_{n^*+m-q+1}(2 + 3^{1/2})^{k-q+1} + B'_{n^*+m-q+1}(2 - 3^{1/2})^{k-q+1}]. \quad (19)$$

By definition of a noncooperative price equilibrium, each firm must have a positive market share. In particular, this must be true for firm q, so that

$$u_{q+1}p_{q+1}^* - u_q p_q^* > \alpha R_1 .$$

The latter condition is verified if and only if condition (17) holds for $n^* + m - q + 1$, which is impossible as long as $q < m + 1$. Consequently, firm q cannot afford a positive market share. As a similar argument applies to firms $k = q + 1,..., k = m$, *exactly* n^* firms, namely, the firms defined by $\{m + 1,..., n^* + m\}$, remain on the market and the corresponding equilibrium prices are provided by (19), where $q = m + 1$, i.e., by (18). Q.E.D.

Acknowledgments

We thank J. Drèze and an anonymous referee for their helpful comments.

References

1. J. Bertrand, Théorie mathématique de la richesse sociale, *J. Savants* (1883), 499–508.
2. E. H. Chamberlin, "The Theory of Monopolistic Competition," 7th ed., Harvard Univ. Press, Cambridge, Mass. 1956.
3. A. A. Cournot, "Recherches sur les principes mathématiques de la théorie des richesses," Librairie des Sciences politiques et sociales, M. Rivière et Cie, Paris, 1838.
4. H. Hotelling, Stability in competition, *Econ. J.* **39** (1929), 41–57.
5. J. Jaskold Gabszewicz and J. P. Vial, Oligopoly "à la Cournot" in a general equilibrium analysis, *J. Econ. Theory* **4** (1972), 381–400.
6. J. Jaskold Gabszewicz and J.-F. Thisse, Price competition, quality and income disparities, *J. Econ. Theory* **20** (1979), 340–359.
7. W. Novshek and H. Sonnenschein, Cournot and Walras equilibrium, *J. Econ. Theory* **19** (1978), 223–266.
8. E. C. Prescott and M. Visscher, Sequential location among firms with foresights, *Bell J. Econ.* **8** (1977), 378–393.